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Free Vibrations of Fluid-containing Spheres

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Abstract. In the paper free vibrations of a spherical shell containing compressed fluid are studied. Its natural frequencies of vibrations are determined under some values of the parameters of the system, influence of geometrical and physical parameters of the system "spherical shell-fluid" on free vibrations of the sphere is studied.

Key Words and Phrases: spherical shell, frequency of free vibrations, potential motion, density

1. Introduction

Shells as elements of machines and constructions are widely used in aircraft and shipbuilding, etc. Therefore, recently the researchers are interested in the issues associated with dynamic behavior of thin-shelled constructions that in working conditions are in contact with external medium. The problems of free vibrations of elastic thin shells contacting with elastic medium and fluid, occupy important place among dynamical contact problems of shell theory. Filled shells may be used in practice for storage and transportation of products. As the problems of strength and life of the shells of tanks are very actual in connection with oil and gas recovery, necessity of storage, transportation and processing of different chemical mixtures. Furthermore, the Earth may be considered as a special shells with a filler.

Frequencies and forms of free vibrations of spherical and cylindrical shells contacting with elastic and liquid medium are studied in [1]-[3]. Approximate simple formulas for calculating frequency and determination of vibration forms of the systems under consideration that restricts the use of the obtained results, as in a number of important cases it excludes the possibility of conducting qualitative analysis of the studied processes, are obtained by approximate methods. These investigations are connected with great difficulties as it is necessary to solve transcendental system of equations.

Free vibrations of a thin-walled shell containing compressible fluid, are studied in [4]-[6]. Under some values of the parameters of the system, its eigenvalues of the frequencies of vibrations were determined, influence of geometrical and physical parameters of the system "cylindrical shell-fluid" on free vibrations of the cylinder is studied.

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In [7], a problem of free vibrations of a thin-walled elastic spherical shell containing an elastic medium with different properties, usually with modulus of elasticity that is significantly less than the elasticity modulus of the shell material, is studied.

Analysis of vibrations of fluid-containing sphere with regard to finite thickness differs from the analysis of a very thin sphere with the fact that loads are not introduced into the equation of motion, and in the equations of motion the terms containing derivatives along the radius, are not ignored. The external load on the shell enters into the boundary conditions. The results may be used when analyzing the tanks subjected to seismic impacts, at transportation and also when studying the Earth vibrations.

In this connection, in this paper we consider free vibrations of a finitely-thickened sphere of radius r_1 and r_2, respectively and filled with compressible fluid. The equation of motion of a spherical shell is disconnected into two parts: the system describing the potential motion, and the equation describing the vortex motion [8].
The first system is of the form:

$$\frac{2(1-\nu)}{1-2\nu}\left(\frac{\partial^2 w}{\partial r^2}+\frac{2}{r}\frac{\partial w}{\partial r}-\frac{2}{r^2}\omega\right)+$$

$$+\frac{1}{r^2}\Delta_0 w+\frac{1}{1-2\nu}\left(\frac{1}{r}\frac{\partial}{\partial r}+\frac{4\nu-3}{r^2}\right)\Delta_0\phi+\lambda^2 w=0$$

$$\frac{1}{1-2\nu}\frac{1}{r}\left(\frac{\partial w}{\partial r}+\frac{4-4\nu}{r}w\right)+\frac{2(1-\nu)}{1-2\nu}\frac{1}{r^2}\Delta_0\phi+\frac{\partial^2\phi}{\partial r^2}+\frac{2}{r}\frac{\partial\phi}{\partial r}+\lambda^2 w=0. \tag{1}$$

In the case under consideration, the conditions on the boundary are:

$$\frac{2G}{1-2\nu}\left|[1-\nu)\frac{\partial w}{\partial r}+\frac{\nu}{r}(2w+\Delta_0\phi]\right|_{r=r_1}=p$$

$$\left|(1-\nu)\frac{\partial w}{\partial r}+\frac{\nu}{r}(2w+\Delta_0\phi)\right|_{r=r_2}=0$$

$$\left|\frac{1}{r}w+\frac{\partial\phi}{\partial r}-\frac{1}{r}\phi\right|_{r=r_1}=0 \tag{2}$$

$$\left|\frac{1}{r}w+\frac{\partial\phi}{\partial r}-\frac{1}{r}\phi\right|_{r=r_2}=0.$$

Here r is the distance from the center of the sphere, w is radial displacement, ϕ is displacement potential, G is shear modulus, ν is Poisson's ratio, q is density of the shell's material.

$$\lambda^2=\frac{q}{G}\omega^2,$$

ω is the frequency of vibrations.

p is the pressure on the inner boundary. Δ_0 is an operator:

$$\Delta_0=\frac{\partial^2}{\partial\theta^2}+ctg\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{3}$$

According to the problem under consideration the solutions are represented by means of spherical harmonics Y_n:

$$w = \omega_n Y_n, \quad \phi = \phi_n Y_n, \;\; p = p_n Y_n. \tag{4}$$

Then

$$\Delta_0 w = -n\,(n+1)\,w, \quad \Delta_0 \phi = -n\,(n+1)\,\phi, \;\; (n = 0, 1, 2)\,,$$

equations (1) and (2) take the form:

$$\left(w_n^{'} + \frac{2}{r} w_n\right)^{'} + \frac{1-2\nu}{2\,(1-2\nu)}\left[\lambda^2 - \frac{n\,(n+1)}{r^2}\right] w_n -$$

$$-\frac{1}{2\,(1-\nu)}\frac{n(n+1)}{r}\left(\phi_n^{'} + \frac{4\nu-3}{r}\phi_n\right) = 0$$

$$\frac{1}{2\,(1-\nu)}\left(\frac{1}{r}w_n^1 + \frac{4-4\nu}{r^2}w_n\right) + \frac{1-2\nu}{2\,(1-\nu)}\Phi_n'' + \frac{1}{r}\Phi_n' \frac{1-2\nu}{(1-\nu)} +$$

$$+\left[\frac{1-2\nu}{2\,(1-\nu)}\lambda^2 - \frac{n\,(n+1)}{r^2}\right] = 0$$

$$\left[(1-\nu)\,w_n' + \frac{2\lambda}{r}w_n - \frac{\nu}{r}n\,(n+1)\,\phi_n\right]_{r=r_1} = p_n \frac{(1-2\nu)}{2G} \tag{5}$$

$$\left[(1-\nu)\,w_n' + \frac{2\nu}{r}w_n - \frac{\nu}{r}n\,(n+1)\,\phi_n\right]_{r=r_2} = 0$$

$$\left(\frac{1}{r}w_n + \phi_n^1 - \frac{1}{r}\phi_n\right)_{r=r_1} = 0$$

$$\left(\frac{1}{r}w_n + \phi_n^1 - \frac{1}{r}\phi_n\right)_{r=r_2} = 0.$$

For the case of potential motion, the pressure of the compressed fluid is determined as follows [10]:

$$p = -\rho \frac{\partial \Pi}{\partial t} \tag{6}$$

where ρ is the fluid density, Π is velocity potential satisfying the equation:

$$a^2 \Delta \Pi = \partial^2 \Pi / \partial t^2 \tag{7}$$

Δ is the Laplace operator, a is the velocity of perturbation propagation.

Radial velocity of the shell and potential of fluid's velocity on the contact surface are connected with the relations:

$$\frac{\partial w}{\partial t} = \frac{\partial \Pi}{\partial r}, \tag{8}$$

where:

$$\Pi = \Pi_\omega i e^{\omega i t}, \;\; w = w_\omega e^{i\omega t}, \;\; p = p_\omega e^{i\omega t}.$$

Taking into account that under vibrations the relations (6) and (8) take the form:

$$\omega w_\omega = \frac{\partial \Pi_\omega}{\partial r} \tag{9}$$

$$p_\omega = \rho\omega\Pi_\omega.$$

Equation (7) turns into the Helmholtz equation. Then the solution of the problem under consideration will have the form:

$$\Pi_{\omega n} = D_n j_n\left(\frac{\omega r}{a}\right), \tag{10}$$

where $j_n\left(\frac{\omega r}{a}\right)$ is Bessel's first kind spherical function. (9), (10) and (4) yield

$$\omega w_n = D_n \frac{\omega}{a} j'\left(\frac{\omega r}{a}\right) \tag{11}$$

$$p_{\omega n} = \rho\omega a j_n\left(\frac{\omega r}{a}\right) w_n / j_n'\left(\frac{\omega r}{a}\right) \tag{12}$$

here $n = 2$, then

$$\frac{j}{j'} = \frac{z\sin z\left(z^2-2\right)+2z^2\cos z}{z\cos z\left(z^2-6\right)-3\sin z\left(z^2-2\right)}.$$

Having integrated the first two equations in (5) within r_1 and r_2 and assuming that the thickness of the solid body of the sphere is small compared with the radius, we get:

$$w_n^1\Big|_{r=r_1}^{r=r_2} + \frac{2}{r}w_n\Big|_1^2 + \frac{1-2\nu}{2(1-\nu)}\left[\lambda^2 - \frac{n(n+1)}{r^2}\right]w_n h -$$

$$-\frac{1}{2(1-\nu)}\frac{n(n+1)}{r}\left(\phi_n|_1^2 + \frac{4\nu-3}{r}\phi_n h\right) = 0$$

$$\frac{1}{2(1-\nu)}\left(\frac{1}{r}\omega_n\Big|_1^2 + \frac{4-4\nu}{r^2}\omega h\right) +$$

$$+\frac{1-2\nu}{2(1-\nu)}\phi_n'\Big|_1^2 + \frac{1-2\nu}{(1-\nu)r}\phi_n'\Big|_1^2 + \left[\frac{1-2\nu}{2(1-\nu)}\lambda^2 - \frac{n(n+1)}{r^2}\right]\phi_n h = 0. \tag{13}$$

Here the values of quantities without indication of the limit, are average.

Connect the deformation in radial direction with inner pressure p, assuming the layer as centrally-symmetric static. Then preserving in the first equation of the system (5) two terms, we have:

$$w_{2n}^1 - w_{1n}^1 + \frac{2}{r}\left(w_{2n} - w_{1n}\right) = 0 \tag{14}$$

or

$$\varepsilon_{2n}^r - \varepsilon_{1n}^r + \frac{2}{r}\left(w_{2n} - w_{1n}\right) = 0 \tag{15}$$

here ε_{in} is deformation.

From the third and fourth equations of the system (5) it follows:

$$(1-\nu)\left(\varepsilon_{2n}^{r}-\varepsilon_{1n}^{r}\right)+\frac{2\nu}{r}\left(w_{2n}-w_{1n}\right)-\frac{\nu}{r}n\left(n+1\right)\left(\phi_{2n}-\phi_{1n}\right)=-\phi_{\omega n}\frac{1-2\nu}{2G} \tag{16}$$

Or, taking into account (15), we get

$$\frac{4\nu-2}{r}\left(w_{2n}-w_{1n}\right)-\frac{\nu}{r}n\left(n+1\right)\left(\phi_{2n}-\phi_{1n}\right)=-\phi_{\omega n}\frac{1-2\nu}{2G} \tag{17}$$

Under the conditions stipulated above, the second equation of the system (5) gives:

$$w_{2n}-w_{1n}+\frac{4-4\nu}{r}w_{n}h=0. \tag{18}$$

From (14) and (18) we have:

$$w_{2}'-w_{1}'=\frac{8\left(1-\nu\right)}{r^{2}}w_{n}h. \tag{19}$$

Substituting (18) in (17), we get:

$$\phi_{2n}-\phi_{1n}=\frac{r}{\nu n\left(n+1\right)}\left[\frac{8\left(1-2\nu\right)\left(1-\nu\right)}{r^{2}}\omega h+\phi_{n}\frac{1-2\nu}{2G}\right]. \tag{20}$$

From the fifth and sixth equation of the system (5) we have

$$\phi_{2n}^{'}-\phi_{1n}^{'}=\frac{1}{r}\left(\phi_{2n}-\phi_{1n}-w_{2n}-w_{1n}\right).$$

Or, using (18) and (20), we get

$$\phi_{2n}^{'}-\phi_{1n}^{'}=\frac{4\left(1-\nu\right)}{r^{2}}\left[\frac{2\left(1-2\nu\right)}{\nu n\left(n+1\right)}+1\right]w_{n}h+\frac{1-2\nu}{\nu n\left(n+1\right)}\frac{P_{\omega n}}{2G}. \tag{21}$$

Substituting (18), (20) and (21) in (13) we get

$$\frac{8\left(1-\nu\right)}{r^{2}}w_{n}h-\frac{8\left(1-\nu\right)}{r^{2}}w_{n}h+\frac{1-2\nu}{2\left(1-\nu\right)}\left[\lambda^{2}-\frac{n\left(n+1\right)}{r^{2}}\right]w_{n}h-$$

$$-\frac{n\left(n+1\right)}{2\left(1-\nu\right)r}\left\{\frac{r}{\nu n\left(n+1\right)}\left[\frac{8\left(1-2\nu\right)\left(1-\nu\right)}{r^{2}}w_{n}h+\frac{1-2\nu}{2G}\phi_{n}\right]+\frac{4\nu-3}{r}P_{\omega n}h\right\}=0$$

$$\frac{1}{2\left(1-\nu\right)}\left[-\frac{4}{r^{2}}\left(1-\nu\right)w_{n}h+4\frac{1-\nu}{r^{2}}w_{n}h\right]+$$

$$+\frac{1-2\nu}{2\left(1-\nu\right)}\left\{\frac{1}{\nu n\left(n+1\right)}\left[8\frac{\left(1-2\nu\right)\left(1-\nu\right)}{r^{2}}w_{n}h+\right.\right.$$

$$\left.\left.+p_{\omega n}\frac{1-2\nu}{2G}\right]+\frac{4\left(1-\nu\right)}{r^{2}}w_{n}h\right\}+\frac{1-\nu}{r}\frac{r}{\nu n\left(n+1\right)}\left[8\frac{\left(1-2\nu\right)\left(1-\nu\right)}{r^{2}}w_{n}h+\right.$$

$$+\frac{1-2\nu}{2G}p_{\omega n}h\Bigg]+\left[\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}-\frac{n\left(n+1\right)}{r^{2}}\right]p_{\omega n}h=0.$$

or

$$\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}w_{n}h-\frac{1-2\nu}{2\left(1-\nu\right)}\frac{n\left(n+1\right)}{r^{2}}w_{n}h-\frac{4\left(1-2\nu\right)}{\nu r^{2}}w_{n}h-$$

$$-\frac{n\left(n+1\right)}{\nu 2\left(1-\nu\right)}-\frac{1-2\nu}{2G}p_{\omega n}+\frac{n\left(n+1\right)}{2\left(1-\nu\right)}\frac{4\nu-3}{r^{2}}p_{\omega n}h=0.$$

$$\left\{8\frac{\left(1-2\nu\right)}{\nu n\left(n+1\right)}\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]+2\left(1-2\nu\right)\right\}\frac{w_{n}h}{r^{2}}+$$

$$+\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]\frac{1-2\nu}{\nu n\left(n+1\right)2G}p_{\omega n}+\left(\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}-\frac{n\left(n+1\right)}{r^{2}}\right)p_{\omega n}h=0. \quad (22)$$

Substituting the expression $p_{\omega n}=\frac{\rho\omega a\,j}{j'}$ from (12) to (22) , we get

$$\left\{\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}-\frac{1-2\nu}{r^{2}}\left[\frac{n\left(n+1\right)}{2\left(1-\nu\right)}+4\right]\right\}w_{n}h+$$

$$+\frac{n\left(n+1\right)\left(4\nu-3\right)}{2\left(1-\nu\right)}\phi_{n}h-\frac{\left(1-2\nu\right)}{2\left(1-\nu\right)2G}\frac{\rho\omega aj}{j'}w_{n}+$$

$$+\left\{4\frac{\left(1-2\nu\right)\left(1-\nu\right)}{\nu n\left(n+1\right)}\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]+1-2\nu\right\}2\frac{w_{n}h}{r^{2}}+$$

$$+\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]\frac{\left(1-2\nu\right)\rho\omega aj}{\nu n\left(n+1\right)2Gj'}+\left[\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}-\frac{n\left(n+1\right)}{r^{2}}\right]p_{\omega n}h$$

$$\left\{\frac{1-2\nu}{2\left(1-\nu\right)}h\lambda^{2}-\frac{1-2\nu}{r^{2}}\left[\frac{n\left(n+1\right)}{2\left(1-\nu\right)}+4\right]h-\frac{\left(1-2\nu\right)}{2\left(1-\nu\right)2G}\frac{\rho\omega aj}{j'}\right\}w_{n}+$$

$$+\frac{n\left(n+1\right)\left(4\nu-3\right)}{2\left(1-\nu\right)}hp_{n\omega}=0$$

$$\left(2\left\{4\frac{\left(1-2\nu\right)\left(1-\nu\right)}{\nu n\left(n+1\right)}\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]+1-2\nu\right\}\frac{h}{r^{2}}+\right.$$

$$\left.+\left[\frac{1-2\nu}{2\left(1-\nu\right)}+1-\nu\right]\frac{\left(1-2\nu\right)}{\nu n\left(n+1\right)}\frac{\rho\omega aj}{2Gj'}\right)w_{n}+$$

$$+\left[\frac{1-2\nu}{2\left(1-\nu\right)}\lambda^{2}-\frac{n\left(n+1\right)}{r^{2}}\right]hp_{\omega n}=0. \quad (23)$$

Accept the following denotation

$$\alpha=\frac{1-2\nu}{2\left(1-\nu\right)}h$$

$$\beta_{1}=-\frac{1-2\nu}{r^{2}}\left[\frac{n\left(n+1\right)}{2\left(1-\nu\right)}+4\right]h-\frac{n\left(n+1\right)\left(1-2\nu\right)}{4\left(1-\nu\right)G}\frac{\rho\,\omega\,a\,j}{j'}$$

$$\gamma_1 = \frac{n\,(n+1)\,(4\nu-3)}{2\,(1-\nu)}h$$

$$\beta_2 = 2\left\{4\frac{(1-2\nu)\,(1-\nu)}{\nu\cdot n\,(n+1)}\left[\frac{1-2\nu}{2\,(1-\nu)}+1-\nu\right]+1-2\nu\right\}\frac{h}{r^2}+$$

$$+\left[\frac{1-2\nu}{2\,(1-\nu)}+1-\nu\right]\frac{(1-2\nu)}{\nu n\,(n+1)}\frac{\rho\,\omega\,a\,j}{2Gj^1}$$

$$\gamma_1 = \frac{n\,(n+1)}{r^2}h.$$

Then (23) has the following form:

$$\begin{cases} \left(\alpha\lambda^2-\beta_1\right)w_n+\gamma_1\phi_n=0 \\ \beta_2 w_n+\left(\alpha\lambda^2-\frac{n(n+1)}{r^2}\right)\phi_n=0 \end{cases} \tag{24}$$

The equation of the system (24) is a system of homogeneous linear equation with respect to variables w_n and ϕ_n. For nontrivial solution, its determinant should equal zero. Then the frequency equation has the form:

$$\alpha^2\lambda^4-(\gamma_2+\beta_1)\,\alpha\lambda^2+\beta_1\gamma_2-\beta_2\gamma_1=0. \tag{25}$$

Here $\gamma_2=\frac{n(n+1)}{r^2}h$.
The solution of the last equation with respect to λ has the form:

$$\lambda^2 = \frac{\alpha\,(\gamma_2+\beta_1)+\sqrt{\alpha\,(\gamma_2+\beta_1)^2+4\,(\beta_2\gamma_1-\beta_1\gamma_2)\,\alpha^2}}{2\alpha^2}. \tag{26}$$

In the case when the sphere is not filled and having denoted by $\lambda=\lambda_0$, $\beta_1=\beta_1^0$, $\beta_2=\beta_2^0$ we get the following dependence:

$$\lambda_0^2 = \lambda^2\frac{\alpha\left(\gamma_2+\beta_1^0\right)+\sqrt{\alpha^2\left(\gamma_2+\beta_1^0\right)^2+4\left(\beta_2^0\gamma_1-\beta_1^0\gamma_2\right)\alpha^2}}{\alpha\,(\gamma_2+\beta_1)+\sqrt{\alpha^2\left(\gamma_2+\beta_1^0\right)^2+4\,(\beta_2\gamma_1-\beta_1\gamma_2)\,\alpha^2}} \tag{27}$$

$$\beta_1^0=\left.\beta_1\right|_{\rho=0};\quad \beta_2^0=\left.\beta_2\right|_{\rho=0}.$$

In this case we use the following denotation:

$$z\,(\omega)=\frac{\omega\cdot r}{a}$$

$$\zeta\,(\omega)=\frac{z\,(\omega)\cdot\left[(z\,(\omega))^2-2\right]\cdot\sin\left(z\,(\omega)\right)+2\,(z\,(\omega))^2\cos\left(z\,(\omega)\right)}{\left[(z\,(\omega))^2-6\right]\cdot z\,(\omega)\cdot\cos\left(z\,(\omega)\right)-3\left[(z\,(\omega))^2-2\right]\cdot\sin\left(z\,(\omega)\right)}$$

$$\beta_1=L\,(\omega)=\frac{-\,(1-2\nu)}{r^2}\cdot\left[\frac{n\cdot(n+1)}{2\cdot(1-\nu)}-4\right]\cdot h+\omega\cdot\frac{\rho\cdot a\cdot(n+1)\cdot(1-2\cdot\nu)}{4\cdot(1-\nu)\cdot G}\cdot\zeta\,(\omega)$$

$$\beta_2 = g(\omega) = \left[\frac{-4(1-2\cdot\nu)^2\cdot(3-2\nu)}{\nu\cdot n\cdot(n+1)} + 1 - \nu\right]\cdot\frac{h}{r^2} +$$

$$+\frac{(3-2\nu)\cdot(1-2\nu)}{4\cdot(1-\nu)\cdot[n(n+1)]\cdot\nu\cdot G}\cdot\rho\,\omega\,a\,\zeta(\omega)$$

$$\beta_1^0 = f(\omega) = \frac{-(1-2\cdot\nu)\cdot h}{r^2}\cdot\left[\frac{n\cdot(n+1)}{2\cdot(1-\nu)} + 4\right] - \frac{n\cdot(n+1)\cdot(1-2\cdot\nu)\,\rho\,\omega\,a\,\zeta(\omega)}{4\cdot(1-\nu)\cdot G}$$

$$\beta_2^0 = j(\omega) = \left[\frac{-4(1-2\cdot\nu)^2\cdot(3-2\nu)}{\nu\cdot n\cdot(n+1)} + 1 - \nu\right]\cdot\frac{h}{r^2} +$$

$$+\frac{(3-4\nu)\cdot(1-2\cdot\nu)}{4\cdot(1-\nu)\cdot n\cdot(n+1)\cdot\nu\cdot G}\cdot\rho\,\omega\,a\,\zeta(\omega).$$

Then (27) takes the form:

$$M(\omega) = \sqrt{\omega^2\cdot\frac{\alpha(\gamma_2+f(\omega)) + \sqrt{[\alpha(\gamma_2+f(\omega))]^2 + 4\cdot\alpha^2(j(\omega)\cdot\gamma_1 - f(\omega)\cdot\gamma_2)}}{(\gamma_2+L(\omega)) + \sqrt{\alpha^2(\gamma_2+L(\omega))^2 + 4\cdot\alpha^2(g(\omega)\cdot\gamma_1 - L(\omega)\cdot\gamma_2)}}}. \quad (28)$$

This expression shows dependence of $m\omega$ frequency of unfilled sphere on the ω frequency of the system.

The graphs of dependences were constructed for different values of parameters. Different parameters of the sphere's thickness were taken into account (fig.1., fig. 2., fig. 3., fig. 4.)

When calculating, the following parameters were taken into account:

$$\gamma = \omega = 0,3;\ n = 2;\ r = 100\ m;\ \rho = 1000\frac{kg}{m^3};\ \ a = 1400\frac{m}{sec.}.$$

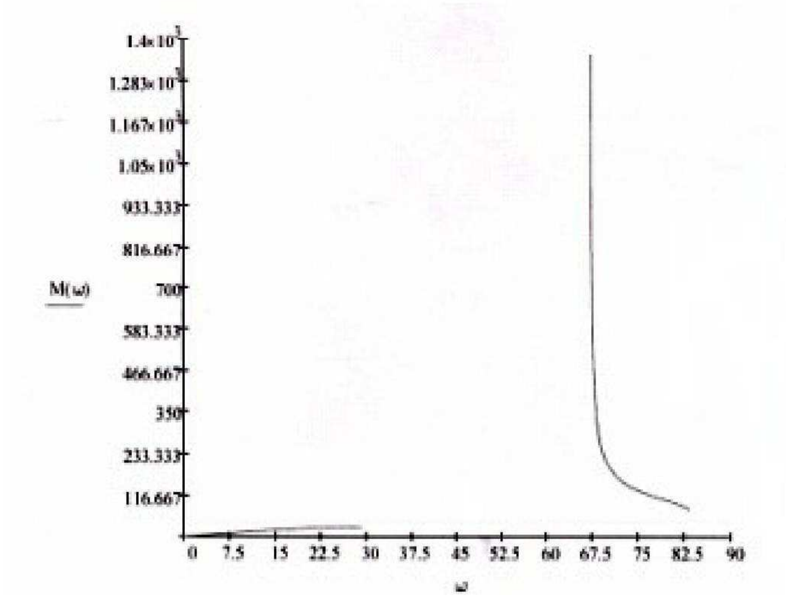

Fig. 1. $h = 0,2m$

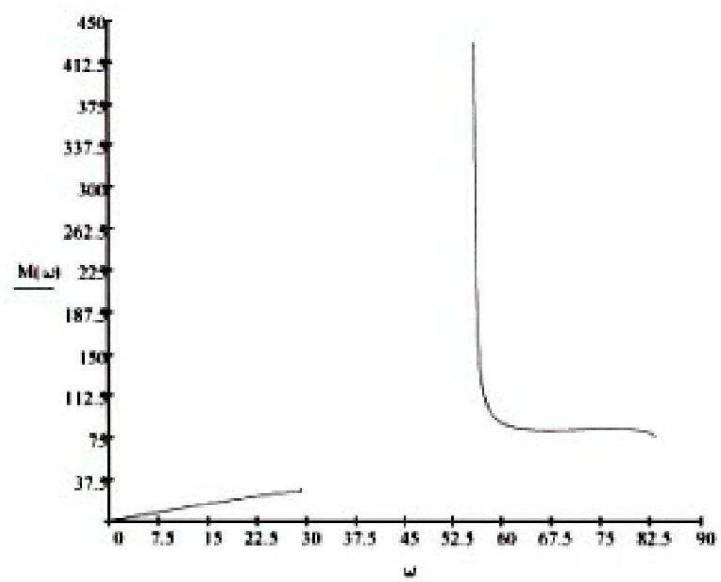

Fig. 2. $h = 0,5m$

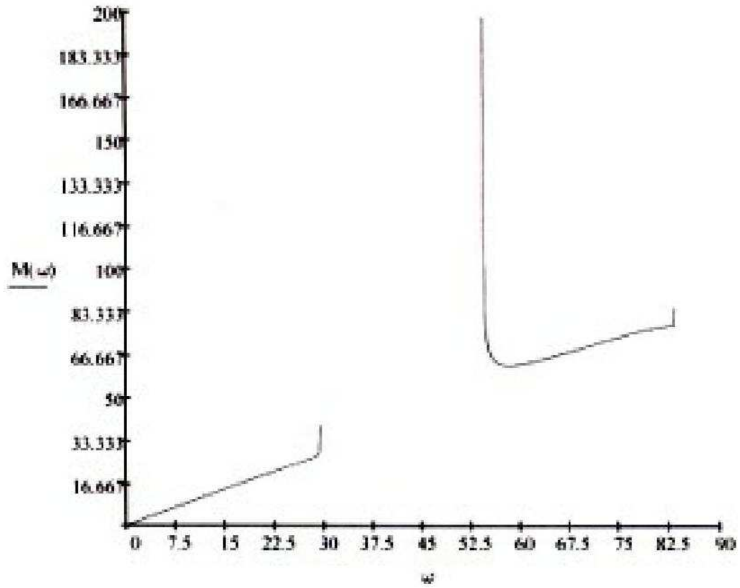

Fig. 3. $h = 1m$

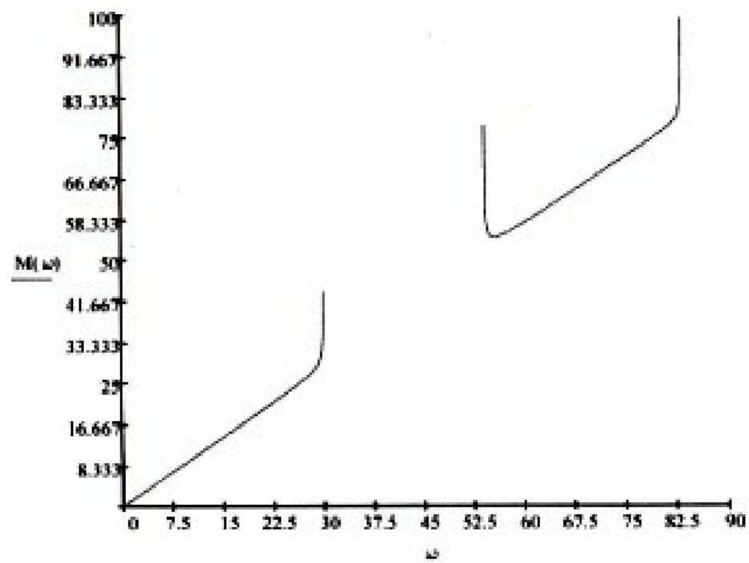

Fig. 4. $h = 5m$

As is seen from the graphs, the frequency of the system for the first mode is linearly connected with the frequency of the empty shell. The system's frequency reaches approximately 30 hertz.

However, for different thicknesses h of the shell for greater thickness, the frequency of the empty shell has the least value (for $h = 0,3m$, $\omega_0 = 37hertz$, for $h = 5m$,

$\omega_0 = 27hertz$). At the end of the mentioned interval, the system's frequency asymptotically approaches to the constant value. Passage to the second mode is accompanied by the "failure" 30 hertz. Then with the same interval the picture of the first mode is repeated. At the ends of the second interval, the system's frequency passes to the constant value.

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