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On the estimation of the informativity of remote ground sensing network of urban air in Baku

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Abstract. In this paper we present a method of estimating the informational value of the rational planning of remote ground sensing network of urban air. To solve this problem we use multi-year actinometric data collected in the city of Baku and its suburbs. This problem is considered from the statistical point of view on the basis of the method of optimal modeling.

Key Words and Phrases: remote sensing, optimal interpolation, network, informativity, urban air

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1. Introduction

In these days of intensive growth of cities, a lot of attention is paid to remote sensing methods of ecological control for urban air. These methods provide most efficient, comprehensive and reliable information about the variations of air pollution on the entire length of the city [1-3].

In this paper we present a method of estimating the informativity of the rational planning of remote ground sensing network of urban air. To solve this problem, we use multi-year actinometric data collected in the city of Baku and its suburbs [2]. This problem is considered from the statistical point of view on the basis of method of optimal modeling [4].

We consider the field of admissible values of observation errors and interpolation. Observation errors are associated with the measurement accuracy and working conditions of aktino photometer. This accuracy is determined by the degree of measurement error. Interpolation errors are associated with the field of maximum allowable angular distances between informative observation points on the daytime sky. We link this distance to the instability of urban air in different parts of the extended area of the city.

Thus, we conduct an assessment for informative overlapping fields of remote sensing of urban air in the city of Baku and its suburbs.

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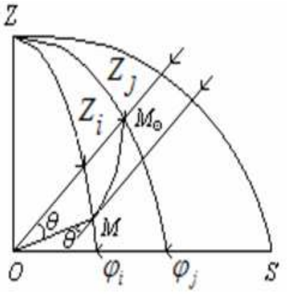
2. Methods of solution

We will describe the distribution of brightness of the daytime sky with the brightness function $f_\lambda(\theta)$, which depends on the angular distance θ between the Sun and the point of observation of the sky and the level of urban air pollution (in what follows the index of the wavelength λ will be omitted) [3].



Figures 1. Geometry of light scattering. M_Θ *and M observed points; Z-zenith, S-south.*

We define the distance between any two points of observation on the celestial sphere in angular units by the cosine formula of spherical geometry (Figure 1):

$$\cos \rho_{i_j} = \cos Z_i \cos Z_j + \sin Z_i \sin Z_j . \cos(\varphi_i - \varphi_j), \tag{1}$$

where Z_i and Z_j are the zenith angles of the observed points on the daytime sky, while φ_i and φ_j are the azimuthal angles between points i and j.

The basic criterion for the problem of the rational planning of remote ground sensing network of urban air is the accuracy of interpolation from points of observation into some other points.

Let us consider the region of space where correlation function $\mu(\rho)$ of the values of $f(\theta)$ can be considered homogeneous and isotropic. At a certain position of the sun, this function will then depend only on the distance ρ between the observed points of the celestial sphere.

Correlation function $\mu(\rho)$ can be determined directly from the observations as follows:

$$\mu(\rho) = \frac{\mu^*(\rho)}{1+\eta^2}, \tag{2}$$

where $\mu^*(\rho)$is the correlation function of the true values of $f(\theta)$, η is the measure of the observational errors, and the correlation function $\mu(\rho)$ tends to $\mu(0) = 1/(1+\eta^2)$ with the decrease in distance ρ.

Let it be required to restore the value of the function $f(\theta_0)$ at any point θ_0 using its measured values in several discrete points by interpolation with the help of the formula

$$f(\theta_0) = \sum_i^n a_i f(\theta_i), \tag{3}$$

where a_i are the weighting coefficients ($\sum_{i=1}^{n} a_i = 1$).

The solution of our problem comes down to the definition of the measure of the errors of interpolation defined as follows [4]:

$$\varepsilon^2 = 1 - \sum_{i=0}^{n} a_i \mu_{0_i}. \tag{4}$$

The coefficients here are defined by the equations

$$\sum_{j=1}^{n} a_j \mu_{i_j} + a_i \eta_i^2 = \mu_{0_i}. \tag{5}$$

In order to meet the requirements of rational placing of remote sensing network, the magnitude of interpolation error will be determined from the condition

$$\varepsilon_m \sim \eta, \tag{6}$$

where ε_m are the maximum allowable values of interpretation errors.

Let's use relations (3) and (5) for the interpolations in the center of a segment, of an equilateral triangle and of a square. Obtained expressions for the calculation of coefficients a_i and measures of interpolation errors ε are presented in Table 1.

Table 1. Formulas for interpolation errors

	weighting coefficients	optimal interpolation
1. on two points	$a_1 = \frac{\mu(\frac{\rho}{2})}{1+\eta^2+\mu(\rho)}$	$\varepsilon_1 = 1 - \frac{2\mu^2(\frac{\rho}{2})}{1+\eta^2+\mu(\rho)}$
2. on three points	$a_2 = \frac{\mu(\frac{\rho}{\sqrt{3}})}{1+\eta^2+2\mu(\rho)}$	$\varepsilon_2 = 1 - \frac{3\mu^2(\frac{\rho}{\sqrt{3}})}{1+\eta^2+2\mu(\rho)}$
3. on four points	$a_3 = \frac{\mu(\frac{\rho}{\sqrt{2}})}{1+\eta^2+2\mu(\rho)+\mu(\rho\sqrt{2})}$	$\varepsilon_3 = 1 - \frac{4\mu^2(\frac{\rho}{\sqrt{2}})}{1+\eta^2+2\mu(\rho)+\mu(\rho\sqrt{2})}$

Urban air in Baku is very unstable in the west of the city, more stable in the south, and middling in the north and the east. Therefore, we use the different correlation functions to describe statistical instability in different parts of the city; see Table 2. In this table, functions $\mu(\rho)$ depend on the parameters $\mu(0)$ or on the measure of the observation errors η as well as on the scale of correlation ρ_0 which, in turn, depends on the area of distribution of urban air.

Table 2. Correlation function of variations of scattering functions of the urban air of Baku in different directions

	Correlation functions	φ_i-φ_j	$\mu(0)$	ρ_0
1. Western direction	$\mu(\rho) = \mu(0).(1\text{-}\rho/\rho_0).\exp(\text{-}\rho/\rho_0)$	80°	0,75	0.85
2. Southern direction	$\mu(\rho) = \mu(0).(1\text{-}\rho/\rho_0)$	120°	0,75	1.18
3. Northern and eastern directions	$\mu(\rho) = \mu(0).\exp(\text{-}\rho/\rho_0)$	160°	0,75	1.37

3. Results of Calculations

Let's consider the dependence of weighting coefficients a_i and interpolation errors ε on the dimensionless parameter ρ/ρ_0.

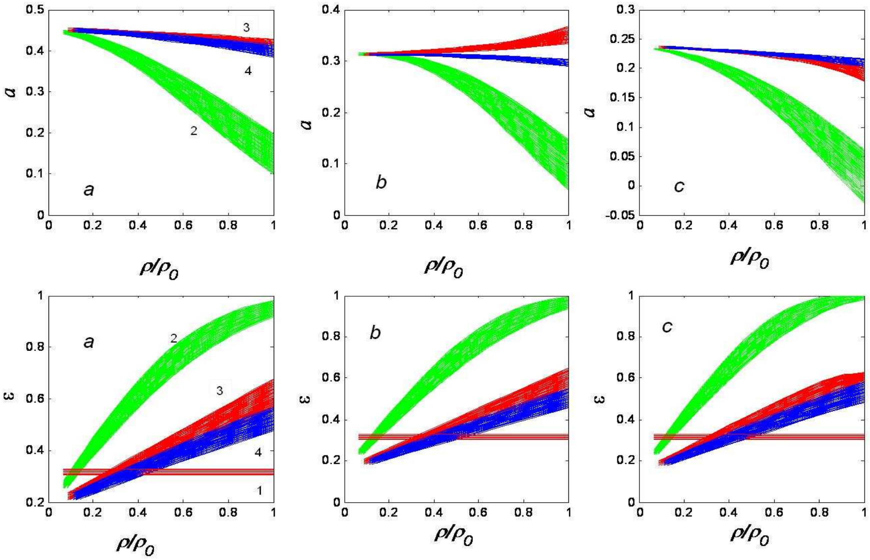


Figure 2. The dependence of the weighting coefficients a_i and errors of interpolation ε on dimensionless parameter $\rho\,/\,\rho_0$ for correlation functions of Table 2: a - on two points, b - on three points and c - on four points of interpolation; curves denote: 1 - the maximum permissible values of the errors of interpolation on condition, 2 - western direction, 3- southern direction, 4 - northern and eastern directions.

In view of the statistical nature of the variability of initial parameters η and ρ_0, we will assume that these parameters can vary within some intervals defined both by casual changes in the measuring device and by the differences in the variations of a sky brightness in different parts of the city. Multi-year actinometrical data [3, 5, 6] show that the changes in η and $\rho_0$can be considered in the intervals

$$\eta = \eta^{\min} + 10\% \cdot \eta^{\min}, \rho_0 = \rho_0^{\max} - 30\% \cdot \rho_0^{\max}. \qquad (7)$$

Results of calculation taking into account (7) are presented graphically in Figure 2. In this Figure, the changes in a_i and ε as well as in the maximum values of interpretation error ε_m occur within certain areas which form corresponding strips.

Table 3 shows the areas of intersection S (km^2) between the fields of interpolation errors values ε and their possible maximum values ε_m depending on the relation ρ/ρ_0. Changes in S (km^2) and in ε characterize the informativity of ground stations providing remote sensing of urban air.

Table 3. Areas of crossing S (km^2) sets of ε and ε_m

$\mu(\rho) = \mu(0)$.(1-ρ/ρ_0).exp(-ρ/ρ_0)	ρ/ρ_0	0,109-0,207	0,1130-0,235	0,121-0,217
	S	1,659.10-3	1,854.10-3	1,681.10^{-3}
$\mu(\rho) = \mu(0)$.(1-ρ/ρ_0)	ρ/ρ_0	0,294-0,562	0,355-0,643	0,329-0,598
	S	5,537.10^{-3}	5,784.10^{-3}	5,481.10^{-3}
$\mu(\rho) = \mu(0)$.exp(-ρ/ρ_0)	ρ/ρ_0	0,352-0,682	0,422-0,756	0,388-0,705
	S	2,87.10^{-3}	4,829.10^{-3}	4,512.10^{-3}

It follows from Figure 2 that the admissible distances between observation points turn out to be considerably bigger in case of interpolation in the centre of an equilateral triangle and a square than in case of interpolation along a straight line. Areas of change S (2) are getting restricted with an increase in the number of points of interpolation. Besides, the difference between three or four points interpolation is not significant. Therefore, three points interpolation is recommended to use when estimating the density of a network in case of optimal interpolation.

Conclusion1. The estimation of informativity of remote ground sensing of urban air in the city of Baku is done with significant difference of the distribution of the brightness of the cloudless sky in different parts of the city, namely, 1) in the west, 2) in the north and the east, and 3) in the south.

2. Three points optimal interpolation is recommended to estimate the density of a network of remote ground sensing of urban air in Baku.

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