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One 3D in the Geometrical Middle Problem in the Non-classical Treatment for one 3D Bianchi Integro-differential Equation with Non-smooth Coefficients

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Abstract. In this paper substantiated for a 3D Bianchi integro-differential equation with non-smooth coefficients a three dimensional in the geometrical middle problem- 3D in the geometrical middle problem with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions three dimensional boundary condition is substantiated classical, in the case if the solution of the problem in the isotropic S. L. Sobolev's space is found. The considered equation as a hyperbolic equation generalizes not only classic equations of mathematical physics (Laplace equation, telegraph equation, string vibration equation) and also many models differential equations (2D and 3D telegraph equation, 2D Bianchi equation, 2D and 3D wave equations and etc.). It is grounded that the in the middle boundary conditions in the classic and non-classic treatment are equivalent to each other. Thus, namely in this paper, the non-classic problem with 3D in the geometrical middle conditions is grounded for a hyperbolic equation of third-order. For simplicity, this was demonstrated for one model case in one of S.L. Sobolev isotropic space $W_p^{(1,1,1)}(G)$.

Key Words and Phrases: 3D in the geometrical middle problem, 3D Bianchi integro-differential equation, 3D mathematical modeling, hyperbolic equations, equation with non-smooth coefficients, equations with dominating mixed derivative.
2010 Mathematics Subject Classifications: 35L25, 35L35

1. Introduction

Hyperbolic equations are attracted for sufficiently adequate description of a great deal of real processes occurring in the nature, engineering and etc. In particular, many processes arising in the theory of fluid filtration in cracked media are described by non-smooth coefficient hyperbolic equations.

Urgency of investigations conducted in this field is explained by appearance of local and non-local problems for non-smooth coefficients equations connected with different applied problems. Such type problems arise for example, while studying the problems of moisture, transfer in soils, heat transfer in heterogeneous media, diffusion of thermal neutrons in inhibitors, simulation of different biological processes, phenomena and etc. [1-3].

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In the present paper, here consider three dimensional in the geometrical middle problem for 3D Bianchi integro-differential equation with non-smooth coefficients. The coefficients in this hyperbolic equation are not necessarily differentiable; therefore, there does not exist a formally adjoint differential equation making a certain sense. For this reason, this question cannot be investigated by the well-known methods using classical integration by parts and Riemann functions or classical-type fundamental solutions. The theme of the present paper, devoted to the investigation in the geometrical middle problem for 3D integro-differential Bianchi equations of hyperbolic type, according to the above-stated is very actual for the solution of theoretical and practical problems. From this point of view, the paper is devoted to the actual problems of applied mathematics and physics.

2. Problem statement

Consider 3D Bianchi integro-differential equation

$$\begin{gathered}(V_{1,1,1}u)(x,y,z) \equiv u_{xyz}(x,y,z) + A_{0,0,0}u(x,y,z) + A_{1,0,0}u_x(x,y,z)+\\ A_{0,1,0}u_y(x,y,z) + A_{0,0,1}u_z(x,y,z) + A_{1,1,0}u_{xy}(x,y,z) + A_{0,1,1}u_{yz}(x,y,z)+\\ +A_{1,0,1}u_{xz}(x,y,z) + \int\limits_{\sqrt{x_0x_1}}^{x}\int\limits_{\sqrt{y_0y_1}}^{y}\int\limits_{\sqrt{z_0z_1}}^{z}[K_{0,0,0}(\tau,\xi,\eta;x,y,z)u(\tau,\xi,\eta)+\\ +K_{1,0,0}(\tau,\xi,\eta;x,y,z)u_x(\tau,\xi,\eta) + K_{0,1,0}(\tau,\xi,\eta;x,y,z)u_y(\tau,\xi,\eta)+\\ +K_{0,0,1}(\tau,\xi,\eta;x,y,z)u_z(\tau,\xi,\eta) + K_{1,1,0}(\tau,\xi,\eta;x,y,z)u_{xy}(\tau,\xi,\eta)+\\ +K_{0,1,1}(\tau,\xi,\eta;x,y,z)u_{yz}(\tau,\xi,\eta)+\\ +K_{1,0,1}(\tau,\xi,\eta;x,y,z)u_{xz}(\tau,\xi,\eta)]\,d\tau d\xi d\eta = \varphi_{1,1,1}(x,y,z),\end{gathered} \tag{1}$$

$(x,y,z) \in G$.

Here $u(x,y,z)$ is a desired function determined on G; $A_{i,j,k} = A_{i,j,k}(x,y,z)$ are the given measurable functions on $G = G_1 \times G_2 \times G_3$, where $G_1 = (x_0,x_1)$, $x_0 \geq 0$, $G_2 = (y_0,y_1)$, $y_0 \geq 0$, $G_3 = (z_0,z_1)$, $z_0 \geq 0$,; $\varphi_{1,1,1}(x,y,z)$ is a given measurable function on G; $K_{i,j,k}(\tau,\xi,\eta;x,y,z)$ are the given measurable functions on $G \times G$.

Equation (1) is a three dimensional Bianchi integro-differential equation with three simple real characteristics $x = const$, $y = const$, $z = const$. Therefore, in some sense we can consider equation (1) as a hyperbolic equation. Equations of the form (1) are used in the modeling of vibration processes [4].

In the present paper 3D Bianchi integro-differential equation (1) is considered in the general case when the coefficients $A_{i,j,k}(x,y,z)$ are non-smooth functions satisfying only the following conditions:

$$A_{0,0,0}(x,y,z) \in L_p(G),$$

$$A_{1,0,0}(x,y,z) \in L^{x,y,z}_{\infty,p,p}(G),$$

$$A_{0,1,0}(x,y,z) \in L^{x,y,z}_{p,\infty,p}(G),$$

$$A_{0,0,1}(x,y,z) \in L^{x,y,z}_{p,p,\infty}(G),$$

$$A_{1,1,0}(x,y,z) \in L^{x,y,z}_{\infty,\infty,p}(G),$$

$$A_{0,1,1}(x,y,z) \in L^{x,y,z}_{p,\infty,\infty}(G),$$
$$A_{1,0,1}(x,y,z) \in L^{x,y,z}_{\infty,p,\infty}(G).$$

In addition, the kernels of integral operators are assumed to satisfy the following conditions: $K_{i,j,k}(\tau,\xi,\eta;x,y,z) \in L_\infty(G \times G)$.

Under these conditions, we'll look for the solution $u(x,y,z)$of equation (1) in S.L.Sobolev isotropic space

$$W^{(1,1,1)}_p(G) \equiv \{u(x,y,z) : D^i_x D^j_y D^m_z u(x,y,z) \in L_p(G),\ i,j,m = \overline{0,1}\},$$

where $1 \leq p \leq \infty$. $D^i_v = \partial^{'}/\partial v^{'}$ is a generalized differentiation operator in S.L.Sobolev sense, D^0_v is an identity transformation operator. We'll define the norm in the space $W^{(1,1,1)}_p(G)$ by the equality

$$\|u\|_{W^{(1,1,1)}_p(G)} = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{m=0}^{1} \|D^i_x D^j_y D^m_z u\|_{L_p(G)}.$$

For 3D Bianchi integro-differential equation (1) we can give the classic form in the geometrical middle boundary conditions in the form :

$$\left\{ \begin{array}{l} u/_{x=\sqrt{x_0 x_1}} = \Phi(y,z), \\ u/_{y=\sqrt{y_0 y_1}} = \Psi(x,z), \\ u/_{z=\sqrt{z_0 z_1}} = g(x,y), \end{array} \right. \tag{2}$$

where $\Phi(y,z)$, $\Psi(x,z)$, and $g(x,y)$ are the given measurable functions on G . It is obvious that in the case of conditions (2), in addition to the conditions

$$\Phi \in W^{(1,1)}_p(G_2 \times G_3) \equiv \left\{ \approx{\Phi}(y,z) : D^j_y D^m_z \approx{\Phi}(y,z) \in L_p(G_2 \times G_3), \quad j,m = \overline{0,1} \right\}$$

$$\Psi \in W^{(1,1)}_p(G_1 \times G_3) \equiv \left\{ \approx{\Psi}(x,z) : D^i_x D^m_z \approx{\Psi}(x,z) \in L_p(G_1 \times G_3), \quad i,m = \overline{0,1} \right\}$$

and

$$g(x,y) \in W^{(1,1)}_p(G_1 \times G_2) \equiv \left\{ \approx{g}(x,y) : D^i_x D^j_y \approx{g}(x,y) \in L_p(G_1 \times G_2),\ i,\ j = \overline{0,1} \right\}$$

the given functions should also satisfy the following agreement conditions:

$$\left\{ \begin{array}{l} \Phi\left(\sqrt{y_0 y_1}\,, z\right) = \Psi\left(\sqrt{x_0 x_1}, z\right), \\ \Phi\left(y, \sqrt{z_0 z_1}\right) = g\left(\sqrt{x_0 x_1}, y\right), \\ \Psi\left(x, \sqrt{z_0 z_1}\right) = g\left(x, \sqrt{y_0 y_1}\right), \end{array} \right. \tag{3}$$

Consider the following non-classical in the geometrical middle boundary conditions :

$$\begin{cases} V_{0,0,0}u \equiv u\left(\sqrt{x_0x_1}, \sqrt{y_0y_1}, \sqrt{z_0z_1}\right) = \varphi_{0,0,0}, \\ (V_{1,0,0}u)(x) \equiv u_x\left(x, \sqrt{y_0y_1}, \sqrt{z_0z_1}\right) = \varphi_{1,0,0}(x), \\ (V_{0,1,0}u)(y) \equiv u_y\left(\sqrt{x_0x_1}, y, \sqrt{z_0z_1}\right) = \varphi_{0,1,0}(y), \\ (V_{0,0,1}u)(z) \equiv u_z\left(\sqrt{x_0x_1}, \sqrt{y_0y_1}, z\right) = \varphi_{0,0,1}(z), \\ (V_{1,1,0}u)(x,y) \equiv u_{xy}\left(x, y, \sqrt{z_0z_1}\right) = \varphi_{1,1,0}(x,y), \\ (V_{0,1,1}u)(y,z) \equiv u_{yz}\left(\sqrt{x_0x_1}, y, z\right) = \varphi_{0,1,1}(y,z), \\ (V_{1,0,1}u)(x,z) \equiv u_{xz}\left(x, \sqrt{y_0y_1}, z\right) = \varphi_{1,0,1}(x,z), \end{cases} \tag{4}$$

where $\varphi_{0,0,0}$ is a given number, and $\varphi_{i,j,k}$ the rest are given functions that satisfy the following conditions:

$$\begin{gathered} \varphi_{1,0,0}(x) \in L_p(G_1), \\ \varphi_{0,1,0}(y) \in L_p(G_2), \\ \varphi_{0,0,1}(z) \in L_p(G_3), \\ \varphi_{1,1,0}(x,y) \in L_p(G_1 \times G_2), \\ \varphi_{0,1,1}(y,z) \in L_p(G_2 \times G_3), \\ \varphi_{1,0,1}(x,z) \in L_p(G_1 \times G_3). \end{gathered}$$

3. Methodology

Therewith, the important principal moment is that the considered equation possesses nonsmooth coefficients satisfying only some p -integrability and boundedness conditions i.e. the considered integro-differential operator $V_{1,1,1}$ has no traditional conjugated operator. In other words, the Riemann function for this equation can't be investigated by the classical method of characteristics. In the papers [5-7] the Riemann function is determined as the solution of an integral equation. This is more natural than the classical way for deriving the Riemann function. The matter is that in the classic variant, for determining the Riemann function, the rigid smooth conditions on the coefficients of the equation are required.

The Riemanns method does not work for hyperbolic equations with non-smooth coefficients. Especially it should be noted that a variety of boundary-value problems for the equations of Bianchi studied in [8-13] and etc.

In the present paper, a method that essentially uses modern methods of the theory of functions and functional analysis is worked out for investigations of such problems. In the main, this method it requested in conformity to integro-differential equations of third-order with simple real characteristics. Notice that, in this paper the considered equation is a generation of many model equations of some processes (for example, 2D and 3D telegraph equation, 2D Bianchi equation, 2D and 3D wave equations and etc).

If the function $u \in W_p^{(1,1,1)}(G)$ is a solution of the classical form 3D in the geometrical middle problem (1), (2), then it is also a solution of problem (1), (4) for $\varphi_{i,j,k}$ defined by the following equalities:

$$\begin{gathered}
\varphi_{0,0,0}=\Phi\left(\sqrt{y_0y_1},\sqrt{z_0z_1}\right)=\Psi\left(\sqrt{x_0x_1},\sqrt{z_0z_1}\right)=g\left(\sqrt{x_0x_1},\sqrt{y_0y_1}\right),\\
\varphi_{1,0,0}(x)=\Psi_x\left(x,\sqrt{z_0z_1}\right)=g_x\left(x,\sqrt{y_0y_1}\right),\\
\varphi_{0,1,0}(y)=g_y\left(\sqrt{x_0x_1},y\right)=\Phi_y\left(y,\sqrt{z_0z_1}\right),\\
\varphi_{0,0,1}(z)=\Phi_z\left(\sqrt{y_0y_1},z\right)=\Psi_z\left(\sqrt{x_0x_1},z\right),\\
\varphi_{1,1,0}(x,y)=g_{xy}\left(x,y\right),\\
\varphi_{0,1,1}(y,z)=\Phi_{yz}\left(y,z\right),\\
\varphi_{1,0,1}(x,z)=\Psi_{xz}\left(x,z\right).
\end{gathered}$$

The inverse one is easily proved. In other words, if the function $u\in W_p^{(1,1,1)}(G)$ is a solution of problem (1), (4), then it is also a solution of problem (1), (2) for the following functions:

$$\Phi(y,z)=\varphi_{0,0,0}+\int\limits_{\sqrt{y_0y_1}}^{y}\varphi_{0,1,0}(\beta)d\beta+\int\limits_{\sqrt{z_0z_1}}^{z}\varphi_{0,0,1}(\gamma)d\gamma+\int\limits_{\sqrt{y_0y_1}}^{y}\int\limits_{\sqrt{z_0z_1}}^{z}\varphi_{0,1,1}(\beta,\gamma)d\beta d\gamma, \quad (5)$$

$$\Psi(x,z)=\varphi_{0,0,0}+\int\limits_{\sqrt{x_0x_1}}^{x}\varphi_{1,0,0}(\alpha)d\alpha+\int\limits_{\sqrt{z_0z_1}}^{z}\varphi_{0,0,1}(\gamma)d\gamma+\int\limits_{\sqrt{x_0x_1}}^{x}\int\limits_{\sqrt{z_0z_1}}^{z}\varphi_{1,0,1}(\alpha,\gamma)d\alpha d\gamma, \quad (6)$$

$$g(x,y)=\varphi_{0,0,0}+\int\limits_{\sqrt{x_0x_1}}^{x}\varphi_{1,0,0}(\alpha)d\alpha+\int\limits_{\sqrt{y_0y_1}}^{y}\varphi_{0,1,0}(\beta)d\beta+\int\limits_{\sqrt{x_0x_1}}^{x}\int\limits_{\sqrt{y_0y_1}}^{y}\varphi_{1,1,0}(\alpha,\beta)d\alpha d\beta. \quad (7)$$

Note that the functions (5)-(7) possess one important property, more exactly, for all $\varphi_{i,j,k}$, the agreement conditions (3) possessing the above-mentioned properties are fulfilled for them automatically. Therefore, equalities (5)-(7) may be considered as a general kind of all the functions $\Phi(y,z),\Psi(x,z)$ and $g(x,y)$ satisfying the agreement conditions (3).

We have thereby proved the following assertion.

Theorem 1. *The 3D in the geometrical middle problem of the form (1), (2) and the non-classical form (1), (4) are equivalent.*

Note that the 3D in the geometrical middle problem in the non-classical treatment (1), (4) can be studied with the use of integral representations of special form for the functions $u\in W_p^{(1,1,1)}(G)$ [14-21]

$$u(x,y,z)=u\left(\sqrt{x_0x_1},\sqrt{y_0y_1},\sqrt{z_0z_1}\right)+\int\limits_{\sqrt{x_0x_1}}^{x}u_\xi\left(\xi,\sqrt{y_0y_1},\sqrt{z_0z_1}\right)d\xi+$$

$$+\int_{\sqrt{y_0y_1}}^{y} u_\eta\left(\sqrt{x_0x_1},\eta,\sqrt{z_0z_1}\right)d\eta+\int_{\sqrt{z_0z_1}}^{z} u_\gamma\left(\sqrt{x_0x_1},\sqrt{y_0y_1},\gamma\right)d\gamma+$$

$$+\int_{\sqrt{x_0x_1}}^{x}\int_{\sqrt{y_0y_1}}^{y} u_{\xi\eta}\left(\xi,\eta,\sqrt{z_0z_1}\right)d\xi d\eta+\int_{\sqrt{y_0y_1}}^{y}\int_{\sqrt{z_0z_1}}^{z} u_{\eta\gamma}\left(\sqrt{x_0x_1},\eta,\gamma\right)d\eta d\gamma+$$

$$+\int_{\sqrt{x_0x_1}}^{x}\int_{\sqrt{z_0z_1}}^{z} u_{\xi\gamma}\left(\xi,\sqrt{y_0y_1},\gamma\right)d\xi d\gamma+\int_{\sqrt{x_0x_1}}^{x}\int_{\sqrt{y_0y_1}}^{y}\int_{\sqrt{z_0z_1}}^{z} u_{\xi\eta\gamma}\left(\xi,\eta,\gamma\right)d\xi d\eta d\gamma.$$

4. Result

So, the classical form 3D in the geometrical middle problem (1), (2) and in non-classical treatment (1), (4) are equivalent in the general case. However, the 3D in the geometrical middle problem (1), (4) is more natural by statement than problem (1), (2). This is connected with the fact that in statement of problem (1), (4) the right sides of boundary conditions don't require additional conditions of agreement type. Note that some boundary-value problems in non-classical treatments for hyperbolic and also pseudoparabolic equations were investigated in the authors papers [22-31].

5. Discussion and conclusions

In this paper a non-classical type 3D in the geometrical middle problem is substantiated for a 3D Bianchi integro-differential equation with non-smooth coefficients and with a third-order dominating derivative. Classic 3D in the middle conditions are reduced to non-classic 3D in the geometrical middle problem by means of integral representations. Such statement of the problem has several advantages: 1) No additional agreement conditions are required in this statement; 2) One can consider this statement as a 3D in the geometrical middle problem formulated in terms of traces in the S.L. Sobolev isotropic space $W_p^{(1,1,1)}(G)$; 3) In this statement the considered 3D Bianchi integro-differential equation is a generalization of many model differential equations of some processes (e.g. 2D and 3D telegraph equation, 2D Bianchi equation, 2D and 3D wave equations and etc.).

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