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Influence of Thickness of Reinforced Cylindrical Shell Filled by Liquid on Free Vibrations

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Abstract. Free oscillations of a reinforced cylindrical shell filled with a liquid are investigated. Based on the technical theory of cylindrical shells, the equations of motion are written down using classical equations in displacements. The fluid motion is potentially described by the wave equation. The liquid moves without detachment from the walls of the cylinders. The fluid pressure is taken into account in the equations of shell motion, and the fluid and shell velocities are equated at the boundaries. Representing the solution in a harmonic form, it converted into a system of transcendental equations. Comparison of the solution of the problem without a liquid with a solution in the presence of a liquid, we find the dependence of the frequency of the system without liquid with the frequency of the system with the liquid. In some values of the system parameters the natural frequencies of the cylinder oscillations are determined.

Key Words and Phrases: cylinder, density of cord filaments, the horizontal movement, the fluid density, volume fraction of cord.

2010 Mathematics Subject Classifications: 539.3

1. Introduction

Circular cylindrical covers emerge with the elements in designs of flying machines and engines, underwater and surface means of transportation, tanks and pipelines, vaulted systems of underwater and underground tunnels and storehouses. Cylindrical covers were widely adopted in the technique. One of the basic spheres of their application are hydraulic systems where such covers are applied in the quality of flexible inserts. Mathematical description of fluctuations of the reinforced covers with fluid is devoted to the set of works [1-7].

One of the most important points in the investigation of fluctuations of covers emerges to be determination of frequencies of free fluctuations that allows to avoid a resonance from external sources of fluctuations or on the contrary to use in need of heat exchange at hashing of liquid products. It is necessary to cancel that the majority considered works are devoted to the elementary special case or to the approached methods.

In work [8], free fluctuations of the thin-walled cylindrical cover containing the compressed liquid are investigated. At some values of parameters of system its own frequencies of fluctuations are defined and influence of geometrical and physical parameters of system a cylindrical cover-liquid on free fluctuation of the cylinder is investigated.

In work [9] frequencies and forms of free fluctuations of the spherical and cylindrical covers contacting to elastic and liquid environments are investigated. Asymptotical methods receive the approached simple formulas for calculation of frequency and definition of the form of fluctuations of the considered systems that limits use of the received results as possibility of carrying out of the qualitative analysis of investigated processes excludes in a number of important cases.

In work [10] the problem of movement of the firm cylinder keeping vertical position under the influence of superficial waves in a liquid is considered. Change of a surface of a liquid is separated to two parts: result of a falling harmonious wave and the indignation caused by presence of the cylinder which thus moves. The problem is accomplished with an operational method. For a finding of the original solution, considering that the image represents a denominator of tabular function, Voltaire's integrated equation of the first sort is used.

2. Problem statement

In the given work free fluctuations of the reinforced cylindrical cover filled with a liquid are investigated. The case of orthotropic covers when cord threads keeps within symmetrically concerning a cover meridian is being considered. The reinforced cover, represents multilayered composite consisting of layers filler and a cord. As the finding of own frequencies of system a cylindrical cover-liquid is connected with the decision of the transcendental equations, frequency of fluctuations of the cover which are not containing a liquid, is expressed through frequency of fluctuations of system in an explicit form that allows both analytically, and graphically to investigate spectra of frequencies of system.

For the description of movement of a cover will use the classical equations in movements [11]. Fluctuations of the liquid filling a cover, are described by the wave equation in cylindrical co-ordinates [12]. On border of contact of a cover with a liquid equality of radial speeds [13] is set.

Thus, fluctuations of considered system is described by the equations:

$$A_{11}u + A_{12}v + A_{13}w = \rho_s h \frac{\partial^2 u}{\partial t^2};$$
$$A_{21}u + A_{22}v + A_{23}w = \rho_s h \frac{\partial^2 v}{\partial t^2};$$
$$A_{31}u + A_{32}v + A_{33}w = -\left(\rho_s h \frac{\partial^2 w}{\partial t^2} + \rho_f \frac{\partial \Phi}{\partial t}\right). \tag{1}$$

Here

$$A_{11} = C_{11}\frac{\partial^2}{\partial x^2} + \frac{C_{66}}{R^2}\frac{\partial^2}{\partial \varphi^2};$$
$$A_{12} = A_{21} = \frac{C_{12} + C_{66}}{R}\frac{\partial^2}{\partial x \partial \varphi};$$

$$A_{13} = A_{31} = \frac{1}{R}\left(C_{12}\frac{\partial}{\partial x}\right);$$

$$A_{22} = \left(C_{66} + \frac{4}{R^2}D_{66}\right)\frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\left(C_{22} + \frac{1}{R^2}D_{22}\right)\frac{\partial^2}{\partial \varphi^2};$$

$$A_{23} = A_{32} = \frac{1}{R}\left(\frac{C_{22}}{R}\frac{\partial}{\partial \varphi} - \frac{1}{R}\left(D_{12} + 4D_{66}\right)\frac{\partial^3}{\partial x^2 \partial \varphi} - \frac{D_{22}}{R^3}\frac{\partial^3}{\partial \varphi^3}\right);$$

$$A_{33} = \frac{1}{R^2}C_{22} + D_{11}\frac{\partial^4}{\partial x^4} + \frac{2}{R^2}\left(D_{12} + 2D_{66}\right)\frac{\partial^4}{\partial x^2 \partial \varphi^2} + \frac{\partial^4}{\partial \varphi^4}. \qquad (2)$$

Here $C_{ik} = hB_{ik}$ $D_{ik} = \frac{h^3}{12}B_{ik}$.

$$B_{11} = B'_{11}\cos^4\theta + 2\left(B'_{12} + 2B'_{66}\right)\sin^2\theta\cos^2\theta + B'_{22}\sin^4\theta B_{22} =$$
$$= B'_{11}\sin^4\theta + 2\left(B'_{12} + 2B'_{66}\right)\sin^2\theta\cos^2\theta + B'_{22}\cos^4\theta B_{12} =$$
$$= B'_{12} + \left(B'_{11} + B'_{22} - 2\left(B'_{12} + 2B'_{66}\right)\right)\sin^2\theta\cos^2\theta(3)B_{66} =$$
$$= B'_{66} + \left(B'_{11} + B'_{22} - 2\left(B'_{12} + 2B'_{66}\right)\right)\sin^2\theta\cos^2\theta, \qquad (3)$$

where

$$B'_{11} = \frac{E_1}{1-\nu_1\nu_2}; \quad B'_{22} = \frac{E_2}{1-\nu_1\nu_2}; \;\; B'_{66} = G; \quad B'_{12} = \frac{\nu_2 E_1}{1-\nu_1\nu_2} = \frac{\nu_1 E_2}{1-\nu_1\nu_2},$$

$E_1,\ E_2,\ \nu_1,\ \nu_2$- composite parameters on the elasticity mainstreams, calculated under formulas [14]:

$$E_1 = E_bV_b + E_m(1-V_b), \frac{1}{E_2} = \frac{V_b}{E_b} + \frac{(1-V_b)}{E_{m,}}, \nu_1 = v_bV_b + v_m\left(1-V_b\right), v_2 = v_2\frac{E_2}{E_1},$$

$$\frac{1}{G} = \frac{V_b}{G_b} + \frac{(1-V_b)}{G_m},$$

where E_b, G_b, v_b- the module the Ship's boy, the module of shift and factor of Puassona, and E_m, $G_{m,}\,v_m$- corresponding parameters filler; V_b- a cord volume fraction.

The density is defined from expression

$$\rho_s = \rho_bV_b + \rho_m\left(1-V_b\right),$$

where ρ_b and ρ_m- density of threads of a cord and filler respectively.

Here following designations are accepted: ρ_s- cover density, ρ_f- liquid density, h- a thickness of a cover, R- radius of a median plane of a cover, B_{ik}- elastic parameters of the generalized law of Guka in cylindrical system of co-ordinates of a cover.

The potential Φ satisfies to the wave equation:

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{\omega^2}{a^2}\Phi = 0. \qquad (4)$$

On border between a liquid and a cover compatibility of movement answers a condition.

$$\frac{\partial w}{\partial t} = -\left.\frac{\partial \Phi}{\partial r}\right|_{r=R}. \tag{5}$$

The decision of system (1) is represented in a kind:

$$\begin{aligned} u &= u_n \cos n\varphi \sin \omega t \cos \frac{\pi x}{l}, \\ \vartheta &= v_n \sin n\varphi \sin \omega t \sin \frac{\pi x}{l}, \\ w &= w_n \cos n\varphi \sin \omega t \sin \frac{\pi x}{l}, \end{aligned} \tag{6}$$

$$\Phi = \Phi_n(r) \cos n\varphi \cos \omega t \sin \frac{\pi x}{l}. \tag{7}$$

Here $\frac{\pi}{l} = k$. Let's substitute (7) in (4), we will receive

$$\Phi_n'' + \frac{1}{r}\Phi_n' + \left(\frac{\omega^2}{a^2} - k^2 - \frac{n^2}{r^2}\right)\Phi_n = 0. \tag{8}$$

In the cylinder the decision of the equation (8) looks as follows [15]:

$$\Phi_n(r) = CJ_n\left(\sqrt{\frac{\omega^2}{a^2} - k^2}\; r\right). \tag{9}$$

Considering (9) in (7)

$$\Phi = CJ_n\left(\sqrt{\frac{\omega^2}{a^2} - k^2}\; r\right) \cos n\varphi \cos \omega t \sin kx. \tag{10}$$

Here C- it is constant, $J_n\left(\sqrt{\frac{\omega^2}{a^2} - k^2}\; r\right)$- function of Bessel of an order n. And applying (6) and (10) in (5), we receive

$$C = -\frac{w_n \omega}{J_n'\left(\sqrt{\frac{\omega^2}{a^2} - k^2} R\right)}. \tag{11}$$

Having substituted (11) in (10), we will receive

$$\Phi = -\frac{w_n \omega J_n\left(\sqrt{\frac{\omega^2}{a^2} - k^2} R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2} - k^2} R\right)} \cos n\varphi \cos \omega t \sin kx. \tag{12}$$

Here a- speed of a sound in a liquid, ω- circular frequency, J_n, $J_n^{'}$- functions of Bessel of an order n. Having substituted (6) and (12) in (1), we have

$$-\left(k^2C_{11}+\frac{n^2}{R^2}C_{66}\right)u_n+\frac{kn}{R}\left(C_{12}+C_{66}\right)\upsilon_n-\frac{k}{R}C_{12}w_n+\rho_s h\omega^2 u_n=0,$$

$$\frac{kn}{R}\left(C_{12}+C_{66}\right)u_n-\left(k^2\left(C_{66}+\frac{4}{R^2}D_{66}\right)-\frac{n^2}{R^2}\left(C_{22}+\frac{1}{R^2}D_{22}\right)+\rho_s h\omega^2\right)\vartheta_n+$$

$$+\frac{1}{R^2}\left(-nC_{22}+k^2n\left(D_{12}+4D_{66}\right)-\frac{n^3}{R}D_{22}\right)w_n=0, \tag{13}$$

$$-\frac{k}{R}C_{12}u_n+\frac{1}{R^2}\left(nC_{22}+k^2n\left(D_{12}+4D_{66}\right)+\frac{n^3}{R^2}D_{22}\right)\upsilon_n+$$

$$+\Bigg(\frac{1}{R^2}C_{22}+k^4D_{11}+\frac{2k^2n^2}{R^2}\left(D_{12}+2D_{66}\right)+\frac{n^4}{R^4}D_{22}-$$

$$-\rho_s h\omega^2+\omega^2\,\rho_f\frac{J_n\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}\Bigg)w_n=0.$$

For simplification it is entered (13) following designations:

$$\left(\alpha_{11}+\rho_s h\omega^2\right)u_n+\alpha_{12}\vartheta_n+\alpha_{13}w_n=0,$$
$$\alpha_{21}u_n+\left(\alpha_{22}+\rho_s h\omega^2\right)\upsilon_n+\alpha_{23}w_n=0, \tag{14}$$
$$\alpha_{31}u_n+\alpha_{32}\vartheta_n+\left(\alpha_{33}-\rho_s h\omega^2+\omega^2\,\rho_f\frac{J_n\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}\right)w_n=0.$$

Here

$$\alpha_{11}=-\left(k^2C_{11}+\frac{n^2}{R^2}C_{66}\right),\alpha_{23}=\frac{kn}{R}\left(C_{12}+C_{66}\right),$$

$$\alpha_{12}=\alpha_{21}=\frac{kn}{R}\left(C_{12}+C_{66}\right),\alpha_{22}=-\left(k^2\left(C_{66}+\frac{4}{R^2}D_{66}\right)+\frac{n^2}{R^2}\left(C_{22}+\frac{1}{R^2}D_{22}\right)\right),$$

$$\alpha_{13}=\alpha_{31}=-\frac{k}{R}C_{12},\alpha_{23}=-\frac{n}{R^2}C_{22}+\frac{k^2n}{R^2}\left(D_{12}+4D_{66}\right)-\frac{n^3}{R^4}D_{22},$$

$$\alpha_{32}=\frac{n}{R^2}C_{22}+\frac{k^2n}{R^2}\left(D_{12}+4D_{66}\right)+\frac{n^3}{R^4}D_{22},$$

$$\alpha_{33}=\frac{1}{R^2}C_{22}+k^4D_{11}+\frac{2k^2n^2}{R^2}\left(D_{12}+2D_{66}\right)+\frac{n^4}{R^4}D_{22}.$$

Let's write out a condition non-triviality decision of system (14) rather $u_n,\ v_n\,,\ w_n$:

$$\begin{vmatrix} \alpha_{11}+\rho_s h\omega^2 & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22}+\rho_s h\omega^2 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33}-\rho_s h\omega^2+\omega^2\,\rho_f\dfrac{J_n\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)} \end{vmatrix}=0.$$

From here we will receive:

$$\left(\alpha_{11}+\rho_s h\omega^2\right)\left(\alpha_{22}+\rho_s h\omega^2\right)\left(\alpha_{33}-\rho_s h\omega^2+\omega^2\,\rho_f\frac{J_n\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}\right)+\alpha_{12}\alpha_{23}\alpha_{31}+$$
$$+\alpha_{13}\alpha_{21}\alpha_{32}-\left(\alpha_{22}+\rho_s h\omega^2\right)\alpha_{13}\alpha_{31}-$$
$$-\alpha_{12}\alpha_{21}\left(\alpha_{33}-\rho_s h\omega^2+\omega^2\,\rho_f\frac{J_n\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}{J_n'\left(\sqrt{\frac{\omega^2}{a^2}-k^2}R\right)}\right)-$$
$$-\alpha_{32}\left(\alpha_{11}+\rho_s h\omega^2\right)\alpha_{23}=0, \tag{15}$$

$$-\rho_s^3h^3\omega^6+\left(-\alpha_{11}-\alpha_{22}+\alpha_{33}\right)\rho_s^2h^2\omega^4+$$
$$+\left(\alpha_{11}\alpha_{33}+\alpha_{22}\alpha_{33}-\alpha_{11}\alpha_{22}-\alpha_{13}\alpha_{31}-\alpha_{32}\alpha_{23}+\alpha_{12}\alpha_{21}\right)\rho h\omega^{\ 2}+$$
$$+\alpha_{11}\alpha_{22}\alpha_{33}+\rho_s^2h^2\omega^6a\rho_f\frac{J_n}{J_n'}+\alpha_{22}\rho_s h\omega^4\rho_f\frac{J_n}{J_n'}+\alpha_{11}\rho_s h\omega^4\rho_f\frac{J_n}{J_n'}+$$
$$+\alpha_{11}\alpha_{22}\omega^2\rho_f\frac{J_n}{J_n'}+\alpha_{12}\alpha_{23}\alpha_{31}+\alpha_{13}\alpha_{21}\alpha_{32}-$$
$$-\alpha_{13}\alpha_{31}\alpha_{22}-\alpha_{11}\alpha_{32}\alpha_{23}-\alpha_{12}\alpha_{21}\alpha_{33}+\alpha_{12}\alpha_{21}\omega^2\rho_f\frac{J_n}{J_n'}=0. \tag{16}$$

The equation (16) represents cubic the equation:

$$\Omega_1^3+A_1\Omega_1^2+A_2\Omega_1+A_3=0. \tag{17}$$

Here

$$\Omega_1=\rho_s h\omega^2, \tag{18}$$
$$A_1=\alpha_{11}+\alpha_{22}-\alpha_{33},$$

$$A_2 = -\alpha_{11}\alpha_{33} - \alpha_{22}\alpha_{33} + \alpha_{11}\alpha_{22} + \alpha_{13}\alpha_{31} + \alpha_{32}\alpha_{23} - \alpha_{12}\alpha_{21},$$

$$A_3 = -\alpha_{11}\alpha_{22}\alpha_{33} - \alpha_{12}\alpha_{23}\alpha_{31} - \alpha_{13}\alpha_{21}\alpha_{32} + \alpha_{13}\alpha_{31}\alpha_{22} + \alpha_{11}\alpha_{32}\alpha_{23} + \alpha_{12}\alpha_{21}\alpha_{33} -$$
$$-\rho_s^2 h^2\omega^6\rho_f\frac{J_n}{J_n'} - \alpha_{22}\rho_s h\omega^4\rho_f\frac{J_n}{J_n'} - \alpha_{11}\rho_s h\omega^4\rho_f\frac{J_n}{J_n'} -$$
$$-\alpha_{11}\alpha_{22}\omega^2\rho_f\frac{J_n}{J_n'} + \alpha_{12}\alpha_{21}\omega^2\rho_f\frac{J_n}{J_n'}.$$

Let's define [14] Ω_1 of (16)

$$\Omega_1 = y - \frac{A_1}{3}, \tag{19}$$

$$y^3 + py + q = 0,$$

$$y_1 = A + B;\quad y_{2,3} = -\frac{A+B}{2} \pm i\frac{A-B}{2}\sqrt{3}, \tag{20}$$

$$A = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}};\quad B = \sqrt[3]{-\frac{q}{2} - \sqrt{Q}};\quad Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{3}\right)^2,$$

$$p = -\frac{A_1^2}{3} + A_2;\quad q = 2\left(\frac{A_1}{3}\right)^3 - \frac{A_1A_2}{3} + A_3.$$

In case of absence of a liquid ($\rho = 0$) the equation (17) will become

$$\left(\Omega_1^0\right)^3 + A_1^0\left(\Omega_1^0\right)^2 + A_2^0\Omega_1^0 + A_0^3 = 0. \tag{21}$$

Here

$$\Omega_1^0 = \rho_s h\left(\omega_0\right)^2, \tag{22}$$

where ω_0 - frequency of free fluctuations of a cover without a liquid.

$$A_1^0 = A_1;\quad A_2^0 = A_2;$$

$$A_3^0 = \alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{12}\alpha_{23}\alpha_{31} + \alpha_{13}\alpha_{21}\alpha_{32} - \alpha_{12}\alpha_{21}\alpha_{33} - \alpha_{12}\alpha_{21}\alpha_{32}.$$

The decision of the equation (21) becomes [15]:

$$\Omega_1^0 = y_1^0 - \frac{A_1^0}{3}, \tag{23}$$

$$y_1^0 = A_0 + B_0;\quad y_{2,3}^0 = -\frac{A_0+B_0}{2} \pm i\frac{A_0-B_0}{2}\sqrt{3}, \tag{24}$$

$$A_0 = \sqrt[3]{-\frac{q_0}{2} + \sqrt{Q_0}};\quad B_0 = \sqrt[3]{-\frac{q_0}{2} - \sqrt{Q_0}};\quad Q_0 = \left(\frac{p_0}{3}\right)^3 + \left(\frac{q_0}{3}\right)^2,$$

$$p_0 = p,\ q_0 = 2\left(\frac{A_1^0}{3}\right)^3 - \frac{A_1^0A_2^0}{3} + A_3^0.$$

Considering (20) in (19):

$$\Omega_1 = A + B - \frac{A_1}{3}, \tag{25}$$

and considering (24) in (23), we receive

$$\Omega_1^0 = A_0 + B_0 - \frac{A_1^0}{3},$$

from here:

$$\frac{\Omega_1^0}{\Omega_1} = \frac{A_0 + B_0 - \frac{A_1^0}{3}}{A + B - \frac{A_1}{3}}.$$

From here

$$\Omega_1^0 = \frac{A_0 + B_0 - \frac{A_1^0}{3}}{A + B - \frac{A_1}{3}}\Omega_1.$$

In other parties from (19) and (22):

$$\omega_0 = \sqrt{\frac{A_0 + B_0 - \frac{A_1^0}{3}}{A + B - \frac{A_1}{3}}\omega}. \quad (26)$$

The formula (26) expresses dependence ω_0 from ω.

The equation (26) connects free frequency of system with free frequency of a cover when there is lack of a liquid. The finding of frequencies of free fluctuations of system is associated with the decision of the transcendental equation (17) at which decision authors often resort to the approached methods, in particular to asymptotic. However, the decision of a return problem allows to build schedules of dependence of frequencies of fluctuations for various fashions of system from frequency of an empty cover that simplifies research, including definition of frequency of free fluctuations of system.

For some values of the system parameters, the dependence of the vibration frequencies for different modes of the system on the frequency of the empty shell is plotted. The influence of the geometric and physical parameters of the system on the free oscillation of the cylinder is studied.

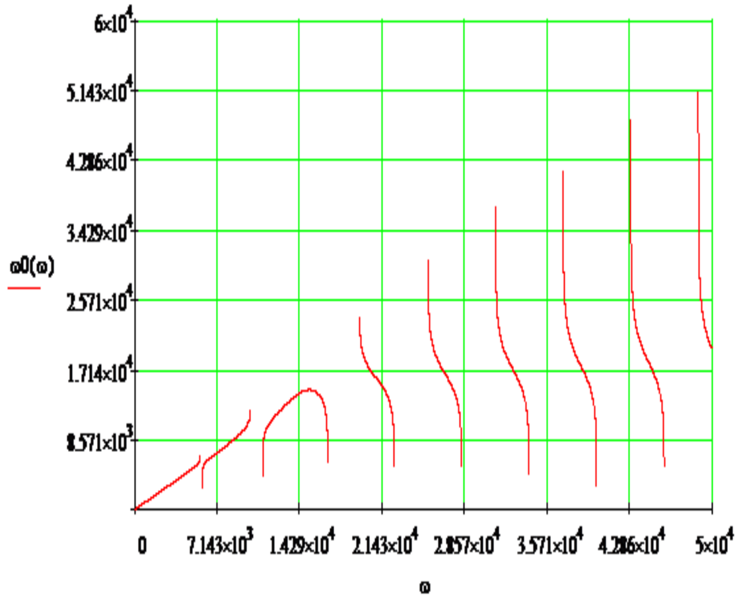

Fig.1
The effect of the thickness of a reinforced cylindrical shell filled with a liquid on free oscillations ($h = 0.08$; h- shell thickness)

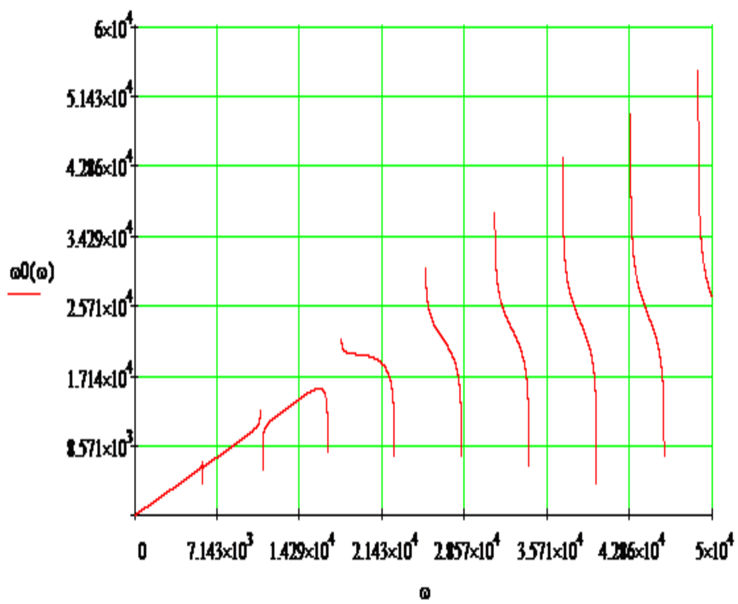

Fig.2
The effect of the thickness of a reinforced cylindrical shell filled with a liquid on free oscillations ($h = 0.5$; h- shell thickness)

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