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On Global Bifurcation from Zero and Infinity in Fourth Order Nonlinear Eigenvalue Problems

N.A. Mustafayeva

Abstract. In this paper we consider nonlinear eigenvalue problems for fourth order ordinary differential equations. We study bifurcation problems from zero and infinity simultaneously for these problems. We prove the existence of two pairs of unbounded continua of solutions corresponding to the usual nodal properties and bifurcating from intervals of the line of trivial solutions and infinity.

Key Words and Phrases: fourth order nonlinear eigenvalue problems, bifurcation point, bifurcation from infinity, bifurcation interval, connected component.
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1. Introduction

We consider the following nonlinear eigenvalue problem

$$\ell y \equiv (py'')'' - (qy')' + r(x)y = \lambda \tau y + h(x, y, y', y'', y''', \lambda),\ x \in (0, l), \tag{1}$$

$$\begin{gathered} y'(0)\cos\alpha - (py'')(0)\sin\alpha = 0, \\ y(0)\cos\beta + Ty(0)\sin\beta = 0, \\ y'(l)\ \cos\gamma + (py'')(l)\ \sin\gamma = 0, \\ y(l)\cos\delta - Ty(l)\sin\delta = 0, \end{gathered} \tag{2}$$

where $\lambda \in \mathbb{R}$ is a spectral parameter, $Ty \equiv (py'')' - qy'$, p is positive, twice continuously differentiable function on $[0, l]$, q is nonnegative, continuously differentiable function on $[0, l]$, r is real-valued continuous function on $[0, l]$, τ is positive continuous function on $[0, l]$ and α, β, γ, $\delta \in [0, \frac{\pi}{2}]$. The nonlinear term h has the form $h = f + g$, where f and g are real-valued continuous functions on $[0, l] \times \mathbb{R}^5$ and there exit $M > 0$ and sufficiently large $c_0 > 0$ such that

$$\left|\frac{f(x,y,s,v,w,\lambda)}{y}\right| \le M,\ x \in [0, l],\ y, s, v, w \in \mathbb{R},\ |y| + |s| + |v| + |w| \le \frac{1}{c_0},\ \lambda \in \mathbb{R}, \tag{3}$$

or

$$\left|\frac{f(x,y,s,v,w,\lambda)}{y}\right| \le M,\ x \in [0, l],\ y, s, v, w \in \mathbb{R},\ |y| + |s| + |v| + |w| \ge c_0,\ \lambda \in \mathbb{R}. \tag{4}$$

Moreover, for any bounded interval $\Lambda \subset \mathbb{R}$

$$g(x,y,s,v,w,\lambda) = o(|y|+|s|+|v|+|w|) \text{ as } |y|+|s|+|v|+|w| \to 0, \tag{5}$$

or

$$g(x,y,s,v,w,\lambda) = o(|y|+|s|+|v|+|w|) \text{ as } |y|+|s|+|v|+|w| \to \infty, \tag{6}$$

uniformly for $x \in [0,l]$ and $\lambda \in \Lambda$.

An important role in nonlinear analysis is played bifurcation theory of nonlinear eigenvalue problems. The bifurcation problem in nonlinear eigenvalue problems occurs in all fields of natural science (see, for example, [4, 5, 9, 10]). Note that, recently have been obtained fundamental results on local and global bifurcation in nonlinear eigenvalue problems for ordinary differential equations (see for example, [1-5, 7-20] and their references).

Similar problems for Sturm-Liouville equation has been considered before by Stuart [19], Toland [20], Rabinowitz [15, 16], Berestycki [7], Schmitt and Smith [18], Rynne [17], Ma and Dai [13], Przybycin [14]. For bifurcation problem from zero in [7, 13-15, 17, 18] the authors prove the existence of two families of global continua of solutions in $\mathbb{R} \times C^1$, corresponding to the usual nodal properties and bifurcating from the eigenvalues and intervals (in $\mathbb{R} \times \{0\}$, which we identify with $\mathbb{R}$) surrounding the eigenvalues of the corresponding linear problem. For bifurcation problem from infinity in [16, 17] show the existence of two families of unbounded continua of solutions bifurcating from the points and intervals in $\mathbb{R} \times \{\infty\}$ and having the usual nodal properties in the neighborhood of these points and intervals.

The nonlinear eigenvalue problem (1)-(2) under the conditions (3) and (5) has been considered by Aliyev [2] (see also [1]), under conditions (4) and (6) has been considered in our recent paper [3]. In these papers for bifurcation problems from zero and infinity we are able to obtain similar results as in the case of nonlinear Sturm-Liouville problems from above.

The purpose of this paper is to study the global bifurcation of nontrivial solutions of problem (1)-(2) in case when conditions (3), (5) and (4), (6) are satisfied simultaneously for f and g, respectively.

2. Preliminary

Let E be the Banach space of all continuously three times differentiable functions on $[0,l]$ which satisfy the conditions (2) and is equipped with its usual norm $||u||_3 = ||u||_\infty + ||u'||_\infty + ||u''||_\infty + ||u'''||_\infty$, where $||u||_\infty = \max\limits_{x\in[0,l]} |u(x)|$.

Let $S = S_1 \cup S_2$, where $S_1 = \{u \in E : u^{(i)}(x) \neq 0, Tu(x) \neq 0,\, x \in [0,l],\, i = 0,\, 1,\, 2\,\}$ and $S_2 = \{u \in E :$ there exists $i_0 \in \{0,\, 1,\, 2\}$ and $x_0 \in (0,1)$ such that $u^{(i_0)}(x_0) = 0$, or $Tu(x_0) = 0$ and if $u(x_0)u''(x_0) = 0$, then $u'(x)Tu(x) < 0$ in a neighborhood of x_0, and if $u'(x_0)Tu(x_0) = 0$, then $u(x)u''(x) < 0$ in a neighborhood of $x_0\}$.

Note that if $u \in S$ then the Jacobian $J = \rho^3 \cos\psi \sin\psi$ of the Prüfer-type transformation

$$\begin{cases} y(x) = \rho(x)\sin\psi(x)\cos\theta(x), \\ y'(x) = \rho(x)\cos\psi(x)\sin\varphi(x), \\ (py'')(x) = \rho(x)\cos\psi(x)\cos\varphi(x), \\ Ty(x) = \rho(x)\sin\psi(x)\sin\theta(x), \end{cases} \tag{7}$$

does not vanish in $(0, l)$ (see [1, 2, 5]).

For each $u \in S$ we define $\rho(u, x)$, $\theta(u, x)$, $\varphi(y, x)$ and $w(u, x)$ to be the continuous functions on $[0, l]$ satisfying

$$\rho(u, x) = u^2(x) + u'^2(x) + (p(x)u''(x))^2 + (Tu(x))^2,$$

$$\theta(u, x) = \operatorname{arctg}\frac{Tu(x)}{u(x)},\ \theta(u, 0) = \beta - \pi/2\,,$$

$$\varphi(u, x) = \operatorname{arctg}\frac{u'(x)}{(pu'')(x)},\ \varphi(u, 0) = \alpha\,,$$

$$w(u, x) = \operatorname{ctg}\psi(u, x) = \frac{u'(x)\cos\theta(u, x)}{u(x)\sin\varphi(u, x)},\ w(u, 0) = \frac{u'(0)\sin\beta}{u(0)\sin\alpha}\,,$$

and $\psi(u, x) \in (0, \pi/2)$, $x \in (0, l)$, in the cases $u(0)u'(0) > 0$; $u(0) = 0$; $u'(0) = 0$ and $u(0)u''(0) > 0$, $\psi(u, x) \in (\pi/2, \pi)$, $x \in (0, l)$, in the cases $u(0)u'(0) < 0$; $u'(0) = 0$ and $u(0)u''(0) < 0$; $u'(0) = u''(0) = 0$, $\beta = \pi/2$ in the case $\psi(u, 0) = 0$ and $\alpha = 0$ in the case $\psi(u, 0) = \pi/2$.

It is apparent that ρ, θ, $\varphi, w : S \times [0, 1] \to \mathbb{R}$ are continuous.

Remark 1. By (7) for each $u \in S$ the function $w(u, x)$ can de determined by one of the following relations

$$\text{a) } w(y, x) = \operatorname{ctg}\psi(y, x) = \frac{(py'')(x)\cos\theta(y, x)}{y(x)\cos\varphi(y, x)},\ w(y, 0) = \frac{(py'')(0)\sin\beta}{y(0)\cos\alpha}\,,$$

$$\text{b) } w(y, x) = \operatorname{ctg}\psi(y, x) = \frac{(py'')(x)\sin\theta(y, x)}{Ty(x)\cos\varphi(y, x)},\ w(y, 0) = -\frac{(py'')(0)\cos\beta}{Ty(0)\cos\alpha}\,,$$

$$\text{c) } w(y, x) = \operatorname{ctg}\psi(y, x) = \frac{y'(x)\sin\theta(y, x)}{Ty(x)\sin\varphi(y, x)},\ w(y, 0) = -\frac{y'(0)\cos\beta}{Ty(0)\sin\alpha}\,.$$

For each $k \in \mathbb{N}$ and each $\nu \in \{+\,, -\}$ let by S_k^ν denote the subset of $y \in S$ such that

1) $\theta(y, l) = (2k-1)\pi/2 - \delta$, where $\delta = \pi/2$ in the case $\psi(y, l) = 0$;

2) $\varphi(y, l) = (k+1)\pi - \gamma$ or $\varphi(u, l) = k\pi - \gamma$ in the case $\psi(y, 0) \in [0, \pi/2)$; $\varphi(y, l) = \pi - \gamma$ for $k = 1$, $\varphi(y, l) = k\pi - \gamma$ or $\varphi(y, l) = (k-1)\pi - \gamma$ for $k \geq 2$ in the case $\psi(y, 0) \in [\pi/2, \pi)$, where $\gamma = 0$ in the case $\psi(y, l) = \pi/2$;

3) for fixed y, as x increases from 0 to l, the function $\theta(y, x)$ ($\varphi(y, x)$) strictly increasing takes values of $m\pi/2$, $m \in \mathbb{Z}$ ($s\pi$, $s \in \mathbb{Z}$); as x decreases, the function $\theta(y, x)$ ($\varphi(y, x)$), strictly decreasing takes values of $m\pi/2$, $m \in \mathbb{Z}$ ($s\pi$, $s \in \mathbb{Z}$);

4) the function $\nu y(x)$ is positive in a deleted neighborhood of $x = 0$.

It follows immediately from the definition of the sets S_k^+, S_k^- and $S_k = S_k^+ \cup S_k^-$, $k \in \mathbb{N}$, that they are disjoint and open in E.

By [2, Theorem 1.2] the eigenvalues of the linear problem

$$\begin{array}{l} \ell(y)(x) = \lambda\tau(x)y(x),\ x \in (0,l), \\ y \in B.C., \end{array} \tag{8}$$

are real and simple and form an infinitely increasing sequence $\{\lambda_k\}_{k=1}^{\infty}$, where by $B.C.$ we denote the set of boundary conditions (2). Moreover, for each $k \in \mathbb{N}$ the eigenfunction $y_k(x)$ corresponding to the eigenvalue λ_k is lies in S_k (therefore $y_k(x)$ has $k-1$ simple nodal zeros in the interval $(0,l)$).

Lemma 1. [2, Lemma 2.2] *If* $(\lambda, y) \in \mathbb{R} \times E$ *is a solution of* (1)-(2) *and* $y \in \partial S_k^{\nu}$, $k \in \mathbb{N}$, $\nu \in \{+\,,\,-\}$, *then* $y \equiv 0$.

Let $\mathcal{C} \subset \mathbb{R} \times E$ denote the set of solutions of problem (1)-(2). We say (λ, ∞) is a bifurcation point (or asymptotic bifurcation point) for problem (1)-(2) if every neighborhood of (λ, ∞) contains solutions of this problem, i.e. there exists a sequence $\{(\lambda_n, u_n)\}_{n=1}^{\infty} \subset \mathcal{C}$ such that $\lambda_n \to \lambda$ and $||u_n||_3 \to +\infty$ as $n \to \infty$ (we add the points $\{(\lambda, \infty) : \lambda \in \mathbb{R}\}$ to space $\mathbb{R} \times E$). Next for any $\lambda \in \mathbb{R}$, we say that a subset $D \subset \mathcal{C}$ meets (λ, ∞) (respectively, $(\lambda, 0)$) if there exists a sequence $\{(\lambda_n, u_n)\}_{n=1}^{\infty} \subset D$ such that $\lambda_n \to \lambda$ and $||u_n||_3 \to +\infty$ (respectively, $||u_n||_3 \to 0$) as $n \to \infty$. Furthermore, we will say that $D \subset \mathcal{C}$ meets (λ, ∞) (respectively, $(\lambda, 0)$) through $\mathbb{R} \times S_k^{\nu}$, $k \in \mathbb{N}$, $\nu \in \{+\,,\,-\}$, if the sequence $\{(\lambda_n, u_n)\}_{n=1}^{\infty} \subset D$ can be chosen so that $u_n \in S_k^{\nu}$ for all $n \in \mathbb{N}$ (in this case we also say that (λ, ∞) (respectively, $(\lambda, 0)$) is a bifurcation point of (1)-(2) with respect to the set $\mathbb{R} \times S_k^{\nu}$). If $I \in \mathbb{R}$ is a bounded interval we say that $D \subset \mathcal{C}$ meets $I \times \{\infty\}$ (respectively, $I \times \{0\}$) if D meets (λ, ∞) (respectively, $(\lambda, 0)$) for some $\lambda \in I$; we define $D \subset \mathcal{C}$ meets $I \times \{\infty\}$ (respectively, $I \times \{0\}$) through $\mathbb{R} \times S_k^{\nu}$, $k \in \mathbb{N}$, $\nu \in \{+\,,\,-\}$, similarly (see [16]).

When the functions f and g satisfies conditions (3) and (5) in [2] show that problem (1)-(2) has a nonempty set of bifurcation points, and if $(\lambda, 0)$ is a bifurcation point of this problem with respect to the set $\mathbb{R} \times S_k^{\nu}$, then $\lambda \in I_k$, where $I_k = [\lambda_k - \frac{M}{\tau_0}, \lambda_k + \frac{M}{\tau_0}]$, $\tau_0 = \min\limits_{x \in [0,\,l]} \tau(x)$.

For $k \in \mathbb{N}$ and $\nu \in \{+\,,\,-\}$ let $\tilde{\mathcal{C}}_k^{\nu}$ denote the union of the connected components $\mathcal{C}_{k,\,\lambda}^{\nu}$ of the solutions set of (1)-(2) under conditions (3) and (5) emanating from bifurcation points $(\lambda, 0) \in I_k \times \{0\}$ with respect to $\mathbb{R} \times S_k^{\nu}$. Let $\mathcal{C}_k^{\nu} = \tilde{\mathcal{C}}_k^{\nu} \cup I_k \times \{0\}$.

Theorem 1. *For each* $k \in \mathbb{N}$ *and each* $\nu \in \{+\,,\,-\}$ *the connected component* $\mathcal{C}_k^{\nu}$ *of* $\mathcal{C}$ *lies in* $(\mathbb{R} \times S_k^{\nu}) \cup (I_k \times \{0\})$ *and is unbounded in* $\mathbb{R} \times E$.

The proof of this theorem is similar to that of [2, Theorem 1.3] by using [2, Theorem 1.2].

In [3] it is prove that the set of asymptotic bifurcation points of problem (1)-(2) under conditions (4) and (6) with respect to the set $\mathbb{R} \times S_k^{\nu}$ is nonempty. Moreover, if (λ, ∞) is an asymptotic bifurcation point for (1)-(2) with respect to the set $\mathbb{R} \times S_k^{\nu}$, then $\lambda \in I_k$.

For each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ we define the set $\mathcal{D}_k^\nu \subset \mathcal{C}$ to be the union of all the components of $\mathcal{C}$ which meet $I_k \times \{\infty\}$ through $\mathbb{R} \times S_k^\nu$. The set $\mathcal{D}_k^\nu$ may not be connected in $\mathbb{R} \times E$, but the set $\mathcal{D}_k^\nu \cup (I_k \times \{\infty\})$ is connected in $\mathbb{R} \times E$.

For any set $A \subset \mathbb{R} \times E$ we let $P_R(A)$ denote the natural projection of A onto $\mathbb{R} \times \{0\}$.

Theorem 2. *For each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ for the set $\mathcal{D}_k^\nu$ at least one of the followings holds:*

(i) *$\mathcal{D}_k^\nu$ meets $I_{k'} \times \{\infty\}$ through $\mathbb{R} \times S_{k'}^{\nu'}$ for some $(k', \nu') \neq (k, \nu)$;*

(ii) *$\mathcal{D}_k^\nu$ meets $\mathcal{R}$ for some $\lambda \in \mathbb{R}$;*

(iii) *$P_R(\mathcal{D}_k^\nu)$ is unbounded.*

In addition, if the union $\mathcal{D}_k = \mathcal{D}_k^+ \cup \mathcal{D}_k^-$ does not satisfy (ii) or (iii) then it must satisfy (i) with $k' \neq k$.

3. Global bifurcation from zero and infinity of solutions of problem (1)-(2)

If conditions (3), (5) and (4), (6) are satisfied simultaneously for f and g, respectively, then we can improve Theorems 1 and 2 as follows.

Theorem 3. *Let the conditions (3)-(6) both hold. Then for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ we have $\mathcal{D}_k^\nu \subset \mathbb{R} \times S_k^\nu$ and alternative (i) of Theorem 2 cannot hold. Furthermore, if $\mathcal{D}_k^\nu$ meets (λ, ∞) for some $\tilde{\lambda} \in \mathbb{R}$, then $\tilde{\lambda} \in I_k$. Similarly, if $\mathcal{C}_k^\nu$ meets $(\tilde{\lambda}, 0)$ for some $\tilde{\lambda} \in \mathbb{R}$, then $\tilde{\lambda} \in I_k$.*

Proof. It follows from Lemma 1 that if conditions (3)-(4) hold, then $\mathcal{C} \cap (\mathbb{R} \times \partial S_k^\nu) = \emptyset$. Hence the sets $\mathcal{C} \cap (\mathbb{R} \times S_k^\nu)$ and $\mathcal{C} \backslash (\mathbb{R} \times S_k^\nu)$ are mutually separated in $\mathbb{R} \times E$ (see [21, Definition 26.4]). Thus by [21, Corollary 26.6] it follows that any connected component of the set $\mathcal{C}$ must be a subset of one or another of the sets $\mathcal{C} \cap (\mathbb{R} \times S_k^\nu)$ and $\mathcal{C} \backslash (\mathbb{R} \times S_k^\nu)$. Since $\mathcal{D}_k^\nu$ is a connected component of $\mathcal{C}$ which intersect $\mathbb{R} \times S_k^\nu$, then $\mathcal{D}_k^\nu$ must be a subset of $\mathbb{R} \times S_k^\nu$, i.e. $\mathcal{D}_k^\nu \subset \mathbb{R} \times S_k^\nu$. But this shows that the alternative (i) of Theorem 2 cannot hold.

Now let $\mathcal{C}_k^\nu$ meets $(\tilde{\lambda}, \infty)$ for some $\tilde{\lambda} \in \mathbb{R}$. Then there exists a sequence $\{(\lambda_{k,n}, y_{k,n})\}_{n=1}^\infty \subset \mathcal{C}_k^\nu$ such that $\lambda_{k,n} \to \tilde{\lambda}$ and $||y_{k,n}||_3 \to \infty$ as $n \to \infty$ and

$$\ell y_{k,n} = \lambda_{k,n} \tau(x) y_{k,n} + f(x, y_{k,n}, y'_{k,n}, \lambda_{k,n}) + g(x, y_{k,n}, y'_{k,n}, \lambda_{k,n}).$$

Let $\lambda \notin I_k$ and

$$\tilde{\delta} = \frac{dist\{\tilde{\lambda}, I_k\}}{2}.$$

Then there exists $n_0 \in \mathbb{N}$ such that

$$\operatorname{dist}\{\lambda_{k,n}, I_k\} > \tilde{\delta}.$$

Obviously, $(\lambda_{k,n}, y_{k,n}) \in \mathcal{C}_k^\nu$ solves the nonlinear problem

$$\begin{cases} \ell y + \varphi_{k,n}(x) y = \lambda \tau(x) y + g(x, y, y', y'', y''', \lambda), \\ y \in B.C., \end{cases} \tag{9}$$

where

$$\varphi_{k,n}(x) = \begin{cases} -\frac{f\left(x, y_{k,n}(x), y_n'(x), y_{k,n}''(x), y_{k,n}'''(x), \lambda_{k,n}\right)}{y_{k,n}(x)} & \text{if} \;\; y_{k,n}(x) \neq 0, \\ 0 & \text{if} \;\; y_{k,n}(x) = 0. \end{cases}$$

By virtue of (5) we have $|\varphi_{k,n}(x)| \leq M$, $n \in \mathbb{N}$, $x \in [0, l]$. Since $y_n(x)$, $n \in \mathbb{N}$, has $k-1$ simple zeros on $(0, l)$ and is bounded on the closed interval $[0, l]$, it follows from [3, Lemma 5.2 and Remark 5.1] that the k-th eigenvalue $\lambda_{k,n}^*$ of the linear problem

$$\begin{cases} \ell y + \varphi_{k,n}(x) y = \lambda \tau(x) y, \;\; x \in (0, l), \\ y \in B.C. \end{cases}$$

lies in I_k. By [11, Ch. 4, § 3, Theorem 3.1] for each $n \in \mathbb{N}$ the point $(\lambda_{k,n}^*, \infty)$ is a unique asymptotic bifurcation point of (9) which corresponds to a continuous branch of solutions that meets this point through $\mathbb{R} \times S_k^\nu$. Hence for each sufficiently large $n > n_0$ we can assign a small $\delta_n > 0$ such that $\delta_n < \tilde{\delta}$ and $|\lambda_{k,n} - \lambda_{k,n}^*| < \delta_n$. Then it follows that $\operatorname{dist}\{\lambda_{k,n}, I_k\} < \tilde{\delta}$, contradicting $\operatorname{dist}\{\lambda_{k,n}, I_k\} > \tilde{\delta}$. Thus $\mathcal{C}_k^\nu$ can only meet $(\tilde{\lambda}, \infty)$ if $\tilde{\lambda} = \lambda_k$. Similarly is proved that $\mathcal{D}_k^\nu$ can only meet $(\tilde{\lambda}, 0)$ if $\tilde{\lambda} = \lambda_k$. The proof of this theorem is complete.

The naturally question arises whether or not $\mathcal{C}_k^\nu$ intersects $\mathcal{D}_k^\nu$. The following examples show that, both cases are possible.

Example 1. *Now we consider the boundary problem*

$$\begin{cases} y^{(4)}(x) = \lambda y(x) + 2y(x) + \lambda \tilde{g}(x, y(x), y'(x), y''(x), y'''(x))\, y(x), \;\; 0 < x < l, \\ y(0) = y''(0) = y(l) = y''(l) = 0, \end{cases} \tag{10}$$

It is obvious that in this case $f(x, y, s, v, w, \lambda) = 2y$ and $g(x, y, s, v, w, \lambda) = \lambda \tilde{g}(x, y, s, v, w)\, y$.

We assume that the function $\tilde{g}$ is satisfied the following conditions:

(i) there exist positive constants K, d and θ such that

$$|\tilde{g}(x, u, s, v, w)| \leq K(|u| + |s| + |v| + |w|)^{-\theta}$$

for all $(x, u, s, v, w) \in [0, l] \times \mathbb{R}^4$ with $|u| + |s| + |v| + |w| \geq d$;

(ii) $\tilde{g}$ is continuous in $[0, l] \times \mathbb{R}^4$ and $f(x, 0, 0, 0, 0) = 0$ for $x \in [0, l]$.

These two conditions ensures that for the function $g(x, u, s, v, w, \lambda) = \lambda \tilde{g}(x, u, s, v, w)$ conditions (4) and (6) both hold.

Then it follows from [3, Example 4.1] that if $\tilde{g}(x, u, s, v, w) \geq 0$ for $(x, u, s, v, w) \in [0, l] \times \mathbb{R}^4$, then $\mathcal{C}_1^\nu \cap \mathcal{D}_1^\nu \neq \emptyset$, and if $\tilde{g}(x, u, s, v, w) \leq 0$ for $(x, u, s, v, w) \in [0, l] \times \mathbb{R}^4$, then $\mathcal{C}_1^\nu \cap \mathcal{D}_1^\nu = \emptyset$.

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Natavan A. Mustafayeva
Ganja State University, AZ2000, Ganja, Azerbaijan
E-mail: natavan1984@gmail.com

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