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On the Parametric Resonance Cases of the System Consisting of the Circular Cylinder and Surrounding Elastic Medium Under Action in the Interior of the Cylinder Time-Harmonic Oscillating Moving Load

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Abstract. The paper studies the parametric resonance cases which appear under the action of the oscillating moving ring load on the interior of the hollow cylinder surrounded by an elastic medium. The axisymmetric stress-strain state is considered and it is assumed that the perfect contact conditions satisfy on the interface between the cylinder and surrounding elastic medium and the equations of motion for the cylinder and surrounding elastic medium are written separately and these equations are exact the so-called 3D equations of the elastodynamics. Numerical results on the interface stresses are presented and according to the analyses of these results, it is established the existence of the parametric resonance in certain values of the moving velocity of the oscillating load.

1. Introduction

The detailed review of the related investigations are given in the papers [1-4] and in the monograph [5] therefore we do not consider here this review again. Nevertheless, we note here some particularities of the recent results which have been obtained with the participation of the author of the present paper. We begin this notation with the paper [1] in which it was shown that under the forced vibration of the system consisting of the hollow cylinder and of the surrounding elastic medium under the action time-harmonic axisymmetric ring forces on the interior of the cylinder the resonance phenomenon does not appear.

In this case, the dependence between the frequency and amplitudes of the quantities characterizing the stress-strain state in the aforementioned system appearing as a result of the time-harmonic ring load has non-monotonic character. In other words, there exists such value of the frequency of the external forces under which the absolute values of the mentioned quantities have their maximum. In other words, there exists such value of the frequency of the external forces under which the absolute values of the mentioned quantities have their maximum. However, in the paper [2] it was established that in the case where on the interior of the cylinder act corresponding non-axisymmetric forces, according to which it was solved the relating three-dimensional problem the noted above dependencies have more complicated character and nevertheless the resonance phenomenon does not observe in the 3D case also. At the same time, the paper [3] establishes that if on the interior of the cylinder the axisymmetric moving constant ring load acts then under certain values moving velocity of this load the resonance type phenomenon takes place and the velocity regarding this case is called the critical velocity.

The question "what kind of the response of the foregoing system to the time-harmonic ring forces acting on the interior of the cylinder appears in the case where these forces move with the constant velocity and this velocity is less than the corresponding critical velocity", is the subject of the investigation of the present paper. As a result of this investigation, it is established that there exist such value of the velocity of the moving load under which the resonance cases appear as a result of the oscillation of the external forces.

2. Formulation of the problem

Consider the aforementioned "hollowcylinder + surrounded elastic medium" system the sketch of which is illustrated in Fig. 1 and assume the thickness of the wall of the cylinder is h and the external radius of the cross section of that is R. Moreover, we assume that on the inner surface of this cylinder normal time-harmonic ring forces act and these forces move along the cylinder axis with constant velocity V. We associate with the central axis of the cylinder the cylindrical system of coordinates $Or\theta z$ and within this framework we attempt to investigate the stress-strain state in the system under consideration with utilizing the following field equations of elastodynamics.

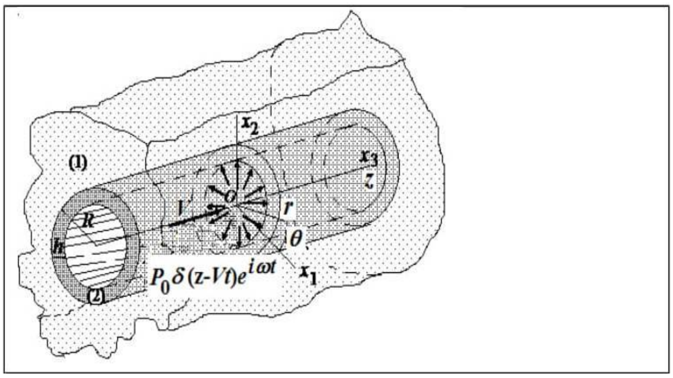

Figure 1: The sketch of the system under consideration and the oscillating moving ring load

Equations of motion:

$$\frac{\partial \sigma_{rr}^{(k)}}{\partial r}+\frac{\partial \sigma_{rz}^{(k)}}{\partial z}+\frac{1}{r}(\sigma_{rr}^{(k)}-\sigma_{\theta\theta}^{(k)})=\rho^{(k)}\frac{\partial^2 u_r^{(k)}}{\partial t^2},\ \frac{\partial \sigma_{rz}^{(k)}}{\partial r}+\frac{\partial \sigma_{zz}^{(k)}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k)}=\rho^{(k)}\frac{\partial^2 u_z^{(k)}}{\partial t^2}. \quad (1)$$

Elasticity relations:

$$\sigma_{nn}^{(k)} = \lambda^{(k)}(\varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)}) + 2\mu^{(k)}\varepsilon_{nn}^{(k)},\ nn = rr;\ \theta\theta;\ zz,\ \sigma_{rz}^{(k)} = 2\mu^{(k)}\varepsilon_{rz}^{(k)}. \tag{2}$$

Strain – displacement relations:

$$\varepsilon_{rr}^{(k)} = \frac{\partial u_r^{(k)}}{\partial r},\ \varepsilon_{\theta\theta}^{(k)} = \frac{u_r^{(k)}}{r},\ \varepsilon_{zz}^{(k)} = \frac{\partial u_z^{(k)}}{\partial z},\ \varepsilon_{rz}^{(k)} = \frac{1}{2}(\frac{\partial u_z^{(k)}}{\partial r} + \frac{\partial u_r^{(k)}}{\partial z}). \tag{3}$$

In equations (1), (2) and (3) the conventional notation of the theory of elasticity is used and through the upper index (k) it is indicated the belonging of the quantities to the cylinder under $k = 2$ and to the surrounding elastic medium under $k = 1$.

Consider also formulation of the corresponding boundary and contact conditions which can be written as follows.

$$\sigma_{rr}^{(2)}\Big|_{r=R-h} = -P_0\delta(z - Vt)e^{i\omega t},\ \sigma_{rz}^{(2)}\Big|_{r=R-h} = 0, \tag{4}$$

$$\sigma_{rr}^{(1)}\Big|_{r=R} = \sigma_{rr}^{(2)}\Big|_{r=R},\ \sigma_{rz}^{(1)}\Big|_{r=R} = \sigma_{rz}^{(2)}\Big|_{r=R},\ u_r^{(1)}\Big|_{r=R} = u_r^{(2)}\Big|_{r=R}.\ u_z^{(1)}\Big|_{r=R} = u_z^{(2)}\Big|_{r=R} \tag{5}$$

$$\left|\sigma_{rr}^{(1)}\right|;\ \left|\sigma_{\theta\theta}^{(1)}\right|;\ \left|\sigma_{zz}^{(1)}\right|;\ \left|\sigma_{rz}^{(1)}\right|;\ \left|u_r^{(1)}\right|;\ \left|u_z^{(1)}\right| \to 0, \quad \text{as } \sqrt{r^2 + z^2} \to \infty. \tag{6}$$

Thus, the investigation of the problem is reduced to the boundary-contact problem (1) – (6) for solution to which the method developed in the papers [1-4] is employed. Now we consider some fragments of the application of this method for the problem under consideration.

3. Method of solution

For solution of the equations (1)-(3). We use the well-known, classical Lame (or Helmholtz) decomposition (see, for instance, the monograph [6] and others listed therein) for solution of the above formulated problem:

$$u_r^{(k)} = \frac{\partial \varPhi^{(k)}}{\partial r} + \frac{\partial^2 \varPsi^{(k)}}{\partial r \partial z},\ u_z^{(k)} = \frac{\partial \Phi^{(k)}}{\partial z} + \frac{\partial^2 \Psi^{(k)}}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi^{(k)}}{\partial r}, \tag{7}$$

where $\Phi^{(k)}$ and $\Psi^{(k)}$ satisfy the following equations:

$$\nabla^2\Phi^{(k)} - \frac{1}{(c_1^{(k)})^2}\frac{\partial^2\Phi^{(k)}}{\partial t^2} = 0\,, \nabla^2\Psi^{(k)} - \frac{1}{(c_2^{(k)})^2}\frac{\partial^2\Psi^{(k)}}{\partial t^2} = 0, \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{8}$$

Here the notation $c_1^{(k)} = \sqrt{(\lambda^{(k)} + \mu^{(k)})/\rho^{(k)}}$ and $c_2^{(k)} = \sqrt{\mu^{(k)}/\rho^{(k)}}$ is used.

We introduce the moving coordinate system

$$r' = r, z' = z - Vt, \tag{9}$$

which moves with the ring load. Representing all the sought values as $g(r,z',t) = \bar{g}(r,z')e^{i\omega t}$ (below, the over bar and upper prime will be omitted) and rewriting the Eq. (8) with the coordinates r' and z' determined in (9), we obtain:

$$\begin{aligned} &\nabla^2\Phi^{(k)} - \frac{1}{(c_1^{(k)})^2}\left(V^2\frac{\partial^2\Phi^{(k)}}{\partial z^2} - 2i\omega V\frac{\partial\Phi^{(k)}}{\partial z} - \omega^2\Phi^{(k)}\right) = 0,\\ &\nabla^2\Psi^{(k)} - \frac{1}{(c_2^{(k)})^2}\left(V^2\frac{\partial^2\Psi^{(k)}}{\partial z^2} - 2i\omega V\frac{\partial\Psi^{(k)}}{\partial z} - \omega^2\Psi^{(k)}\right) = 0. \end{aligned} \tag{10}$$

During the foregoing transformations, the first condition in (4) transforms to the following one:

$$\sigma_{rr}^{(2)}\Big|_{r=R-h} = -P_0\delta(z), \tag{11}$$

but the other relations and conditions in (1) – (6) remain valid for the amplitudes of the sought values.

Below we will use the dimensionless coordinates $\bar{r} = r/h$ and $\bar{z} = z/h$ instead of the coordinates r and z, respectively and the over-bar in $\bar{r}$ and $\bar{z}$ will be omitted.

Further, we employ the exponential Fourier transform $f_F = \int_{-\infty}^{+\infty} f(z)e^{isz}dz$, according to which, the functions $\Phi^{(k)}$ and $\Psi^{(k)}$, and the amplitudes of the sought values can be presented as follows:

$$\left\{\Phi^{(k)};\Psi^{(k)};u_z^{(k)};u_r^{(k)};\sigma_{nn}^{(k)};\sigma_{rz}^{(k)};\varepsilon_{nn}^{(k)};\varepsilon_{rz}^{(k)}\right\}(r,z) =$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\Phi_F^{(k)};\Psi_F^{(k)};u_{zF}^{(k)};u_{rF}^{(k)};\sigma_{nnF}^{(k)};\sigma_{rzF}^{(k)};\varepsilon_{nnF}^{(k)};\varepsilon_{rzF}^{(k)}\right\}(r,s)e^{-isz}ds, nn = rr;\ \theta\theta;\ zz. \tag{12}$$

Substituting the expressions in (12) into the foregoing equations, relations and contact and boundary conditions, we obtain the corresponding ones for the Fourier transformations of the sought values. After this transform the relation (2), the first and second relation in (3), the second condition in (4) and all the conditions in (5) and (6) also remain valid for their Fourier transforms. Nevertheless, the third and fourth relation in (3), the condition (11) and the relations in (7) transform to the following ones:

$$\begin{aligned} &\varepsilon_{zzF}^{(k)} = isu_{zF}^{(k)}, \varepsilon_{rzF}^{(k)} = \tfrac{1}{2}\left(\tfrac{\partial u_{zF}^{(k)}}{\partial r} - isu_{rF}^{(k)}\right), \sigma_{rrF}^{(2)}\Big|_{r=R-h} = -P_0 u_{rF}^{(k)} = \frac{\partial\Phi_F^{(k)}}{\partial r} - is\frac{\partial\Psi_F^{(k)}}{\partial r},\\ &u_z^{(k)} = -is\Phi_F^{(k)} + \frac{\partial^2\Psi_F^{(k)}}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi_F^{(k)}}{\partial r} \end{aligned} \tag{13}$$

where, according to (8), the functions $\Phi_F^{(k)}$ and $\Psi_F^{(k)}$ are determined from the equations:

$$\begin{aligned} &\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(s^2 - \frac{W^2(c_2^{(2)})^2}{(c_1^{(k)})^2}\right)\right]\Phi_F^{(k)} = 0,\\ &\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(s^2 - \frac{W^2(c_2^{(2)})^2}{(c_2^{(k)})^2}\right)\right]\Psi_F^{(k)} = 0, \end{aligned} \tag{14}$$

where

$$W = \Omega - sc, \ \Omega = \frac{\omega h}{c_2^{(2)}}, \ c = \frac{V}{c_2^{(2)}}. \tag{15}$$

Taking into consideration the conditions in (6), the solution to the equations in (14) are found as follows:

$$\begin{gathered} \Phi_F^{(2)} = A_1 H_0^{(1)}(r_1) + A_2 H_0^{(2)}(r_1), \Psi_F^{(2)} = B_1 H_0^{(1)}(r_2) + B_2 H_0^{(2)}(r_2), \\ \Phi_F^{(2)} = C_2 H_0^{(2)}(r_{11}), \Psi_F^{(2)} = D_2 H_0^{(2)}(r_{21}), \end{gathered} \tag{16}$$

where $H_0^{(1)}(x)$ and $H_0^{(2)}(x)$ are the Hankel functions of the first and second kinds, respectively and

$$\begin{gathered} r_1 = r\sqrt{W^2\delta_1^2 - s^2}, \delta_1 = \frac{c_2^{(2)}}{c_1^{(2)}}, \ r_2 = r\sqrt{W^2 - s^2}, \\ r_{11} = r\sqrt{W_1^2\delta_2^2 - s^2}, \ W_1 = W\frac{c_2^{(2)}}{c_2^{(1)}}, r_{21} = r\sqrt{W_1^2 - s^2}. \end{gathered} \tag{17}$$

Substituting the expressions in (17) into (13) and the Fourier transforms of the expressions in (2) it is obtained the analytic expressions for the Fourier transforms of the sought values which contain the unknown constants A_1, A_2, B_1, B_2, C_2 and D_2. Using the Fourier transforms of the contact and boundary conditions (4) and (5) the system of algebraic equations are obtained for these unknowns. Thus, solving this system of equations the Fourier transforms of the sought values are determined completely.

The originals of the aforementioned transforms are determined numerically the algorithm for which are proposed and discussed in the papers [1-5]. Therefore we do not consider here the algorithm and their testing which are used under obtaining numerical results which are discussed below.

4. Numerical results and their discussions

First of all, we note that the numerical results which will be considered below are obtained in the following three cases.

Case 1.

$$E^{(1)} \big/ E^{(2)} = 0.35, \ \rho^{(1)} \big/ \rho^{(2)} = 0.1, \ \nu^{(1)} = \nu^{(2)} = 0.25. \tag{18}$$

Case 2.

$$E^{(1)} \big/ E^{(2)} = 0.05, \ \rho^{(1)} \big/ \rho^{(2)} = 0.01, \ \nu^{(1)} = \nu^{(2)} = 0.25. \tag{19}$$

Case 3.

$$E^{(1)} \big/ E^{(2)} = 0.5, \ \rho^{(1)} \big/ \rho^{(2)} = 0.5, \ \nu^{(1)} = \nu^{(2)} = 0.3. \tag{20}$$

We consider of the frequency response of the interface normal stress

$$\sigma_{rr}(z) = \sigma_{rr}^{(1)}(R,z) = \sigma_{rr}^{(2)}(R,z) \tag{21}$$

in the foregoing cases (18)-(20) for various values of dimensionless moving velocity $c = V/c_2^{(2)}$. The graphs of these responses are illustrated in Figs. 2, 3 and 4 for the cases (18), (19) and (20), respectively.

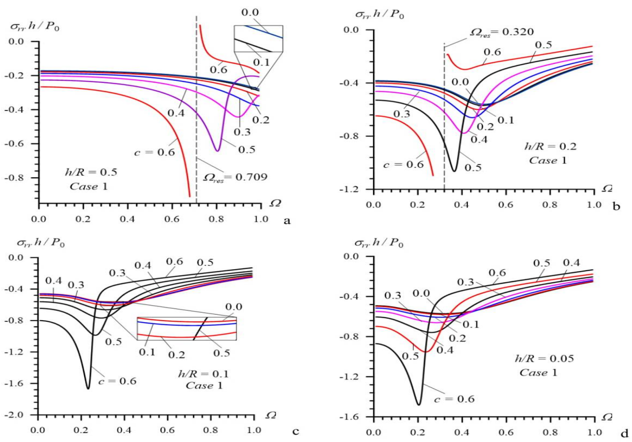

Figure 2: Frequency response of the interface normal stress σ_{rr} obtained for various values of the load moving velocity under $h/R = 0.5$ (a), 0.2 (b), 0.1 (c) and 0.05 (d) in Case 1

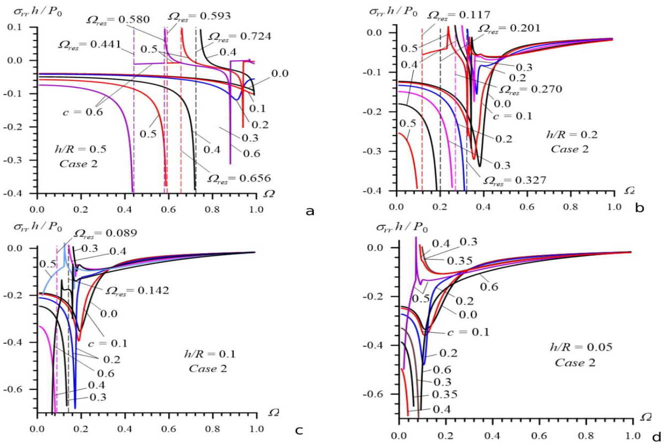

Figure 3: Graphs indicated in Fig. 2 caption and constructed in Case 2

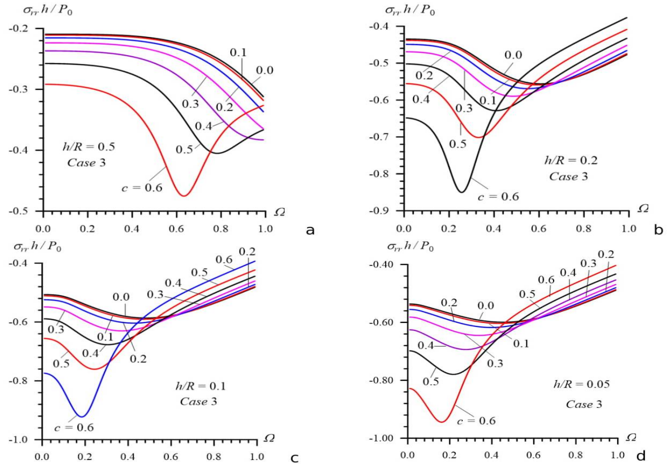

Figure 4: Graphs indicated in Fig. 2 caption and constructed in Case 3

It follows from Figs.2, 3 and 4 that in the all cases under consideration (except the case where $h/R = 0.5$ and $0 \leq c \leq 0.3$ in Case 1 and Case 3, and $0 \leq c \leq 0.1$ in Case 3 under which the absolute values of the stress increase with Ω in the considered change range) the frequency responses have non-monotonic character, i.e. there are such values of Ω (denote this value of Ω by $\Omega*$) before which the absolute value of the stress σ_{rr} becomes maximum and this maximum increases with the moving velocity of the ring load. At the same time, it follows from the results that the values of $\Omega*$ decrease monotonically with c. Moreover, Figs. 2a, 2b, 3a, 3b, 3c and 3d show that there may be cases where an increase in the values of c leads to resonance cases. Such resonance cases, and the corresponding resonance frequencies are indicated in these figures.

The above-noted resonances can be estimated as a parametric resonance and as a parameter it can be taken as the load moving velocity. Consequently, under oscillating moving load action of the ring load, resonance type accidents appear not only under critical moving velocities of this load but also under the foregoing type of parametric resonances. Analyses of the foregoing results also show that the absolute maximum values of the stress under consideration increase with decreasing of the ratio h/R. Moreover, comparison of the results obtained for Case 1, Case 2 and Case 3 with each other shows that the responses of the interface normal stress to the moving velocity of the ring load and its vibration depend not only on the values of this velocity and frequency, but also depend significantly on the ratio of the mechanical properties of the selected pairs of materials, as indicated in (18) – (20) for the hollow cylinder and surrounding elastic medium. At the same time, the latter dependence has not only quantitative, but also qualitative character.

5. Conclusions

Thus, in the present paper the parametric resonance of the system consisting of the hollow cylinder and surrounding elastic medium under action of the time-harmonic oscillating moving ring load acting in the interior of the cylinder is studied. The study is made within the scope of the exact equations and relations of the elastodynamics in the axisymmetric stress-state case. It is described the problem formulation and solution method for this problem.

Numerical results are presented for certain cases which are determined with the ratio of the mechanical constants of the constituents. As a result of the analyses of these results, it is established that there exist the cases under which in the certain values of the velocity of the moving load the oscillation of the moving load causes the resonance of the bi-material elastic system under consideration. The appearance of the resonance cases depends also on the ratio of the cylinder thickness to the cylinder external radius.

The obtained results and their discussions show that the investigations of the problem under consideration have not only theoretical but also the application significance under construction of underground structures. Therefore, it can be concluded that it is necessary to develop such type investigations for the other related problems.

Finally, we note that the results obtained in the present paper have been presented in the 6-th International Conference on Control and Optimization with Industrial Application and the related summary has been published in [7].

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