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Eigensubspaces of Resonancing Endomorphisms of Algebra of Convergent Power Series

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Abstract. In this work we investigate the eigenvalues and eigensubspaces of algebra of convergent power series $\sum_n$ of n variables $z = (z_1, \ldots, z_n)$. We introduce the notion resonancing endomorphisms and for the algebras $\sum_2$ we determine the eigenvalues of resonancing endomorphisms, also describe their corresponding eigensubspaces.

Key Words and Phrases: resonancing eigenvalue, resonancing monom, eigensubspaces, endomorphism, uniform algebra.

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1. Introduction

In [2] Kamowitz was considered the weighted composition operator T on the disc-algebra (i.e. the algebra of continuous functions on the closed unit disc and analytic in the interior of its) and was determined its spectrum in the case when T is compact. In [3] we have more generally results inclusion multidimensional cases.

In [3] was considered the weighted composition operators actings on uniform spaces of analytic functions, which induced by the compressly mappings on the bounded domains $D \subset C^n$ $(n \geq 1)$ and was determined its spectrum. Another words, if D is a bounded domain and $\varphi : \overline{D} \to D$ is holomorphic mapping (where $\overline{D}$ denote closure of D), then in [3] was considered the operators of the form $T : X \to X,\ f \to u \cdot f \circ \varphi$, for every $f \in X$, where $u \in X$ is fixed function and X is Banach-A(D) module, which is uniform subspace of space of holomorphic functions on D equipped with uniform topology. It is well known the mapping φ has a unique fixed point in D. In [3] was shown the spectrum of operator T is equal to semigroup induced by eigenvalues of linear part of φ at the fixed point. Since these operators are compacts, then every eigensubspace corresponding to nonzero eigenvalue has finite dimensions. But from method of [3] we know about dimensions of eigensubspaces, if only case when differential of mapping φ at the fixed point has differently, nonzero and multiplicativly independent eigenvalues, and in this case corresponding eigensubspace has dimension 1.

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In this work we will avoid the results of [3], in the above mentioned case, calculate directly the spectrum of operator T and additionally we will have all informations about eigensubspaces of T and theirs dimensions, when $n = 2$. We may assume, as so as [4], weighted function u is identity, and domain of φ, which induced the weighted endomorphism T contains the origin of coordinate and it is fixed point for mapping φ. Since in this case between eigenvalues and eigensubspaces of operator T and eigenvalues and eigensubspaces of endomorphism of algebra of formal (or convergent) series there are bijective mapping (see [4]), so we begin investigate last problems.

Investigation of spectral properties (for example, spectrum, eigenvalues, eigensubspaces and so) of endomorphisms, also weighted endomorphisms on different algebras (for example, on the uniform algebras, especially on the function algebras with analytic structure, etc), usually leads to investigation these problems on the algebras formally convergent power series (instance , in the case algebra of analytic functions, we have the algebra of germs of functions at the fixed points, etc). Moreover, in many cases studying some algebraic and spectral properties of endomorphisms, or weighted endomorphisms induced by compression mappings (for example, see [3]), or more generally, by the mappings which have fixed points, in some sense (for example, in the Denjoy-Wolff sense fixed point, and so) on the function algebras with analytic structure, again leads to studying endomorphisms of above mentioned algebras. Especially, on the uniform algebras spectrum of the compact, or quazi-compact weighted endomorphisms described by the eigennumbers of linear part of endomorphism at the origin, which modules less than 1 (see [3]). So, in this work we will assume that modules of eigennumbers of the linear part of mapping (which induced the given endomorphism) on initial point of coordinate system less than 1.

2. Eigensubspaces of nonresonancing endomorphisms on the algebra $\sum_n$

Let $\sum_n$ be the algebra of convergent power series of $n \geq 1$ variables $z = (z_1, \dots, z_n)$. In this algebra we will determine eigenvalues of endomorphism generated by mapping Φ which modules of all eigenvalues of its linear part Φ_1 at the origin less than 1, nonzero, differently and nonresonancing (if, $\alpha_1, \dots, \alpha_n$ are eigenvalues of Φ_1, then α_s is called resonancing eigenvalue, if $\alpha_s = \alpha^m = \alpha_1^{m_1} \dots\ \alpha_n^{m_n}$, where, $m_i \geq 0$, $\sum_{i=1}^n m_i \geq 2$; for any resonancing eigenvalue $\alpha_s = \alpha^m$ corresponding resonancing vector-monom $z^m e_s$, where e_s is basic vector and $z^m = z_1^{m_1} \dots z_n^{m_n}$; if, between eigenvalues $\alpha_1, \dots, \alpha_n$ there is resonancing conditions, then the endomorphism is called resonancing endomorphism, otherwise is called nonresonancig endomorphism). It is clear that in this case by Puancare's theorem (see [1]) we may by diffeomorphic transformation of coordinates the mapping Φ reduced to its linear part Φ_1, which has diagonal form. Let Φ_1 has a form $\Phi_1 = diag\ (\alpha_1, \dots, \alpha_n)$. Sufficiently we calculate the eigenvalues of the endomorphism of the form $T\ : f \mapsto Tf = f \circ \Phi_1$, where f is convergence power series. If, f has the form $f(z) = \sum_k a_k z^k$, where k=$(k_1, \dots .k_n)$ is multindex, then we have

$$Tf(z) = f(\Phi_1(z) = \sum_k a_k \Phi_1^k(z) = \sum_{k_1 \dots . k_n} a_{k_1\ \ , \dots k_n} (\alpha_1 z_1)^{k_1} \dots . (\alpha_n z_n)^{k_n}\ .$$

We consider the eigenvalue problem:

$$Tf\,(z) = \lambda f\,(z)\,;\; \sum_{k_1\ldots.k_n} a_{k_1\ \ ,\ldots k_n}\alpha_1^{k_1}\ldots.\alpha_n^{k_n}.z_1^{k_1}\ldots z_n^{k_n} = \lambda \sum_{k_1\ldots.k_n} a_{k_1\ \ ,\ldots k_n} z_1^{k_1}\ldots z_n^{k_n}.$$

In this equation if, some coefficient $a_{k_1\ ..,\ k_n}$ is nonzero ($a_{k_1\ ..,\ k_n} \neq\ 0$), then we have that:

$$\lambda = \alpha_1^{k_1}\ldots\ \alpha_n^{k_n}.$$

Consequently, every eigenvalue of endomorphism T has the form

$$\lambda_{k=}\alpha_1^{k_1}\ldots\ \alpha_n^{k_n}$$

Since every function $z_1^{k_1}\ldots z_n^{k_n}$ is eigenfunction for the endomorphism T corresponding eigenvalue $\lambda_{k=}\alpha_1^{k_1}\ldots\ \alpha_n^{k_n}$, so we conclude that all numbers of the form $\lambda_{k=}\alpha_1^{k_1}\ldots\ \alpha_n^{k_n}$ are eigenvalues of the endomorphism T. In this way we proved next theorem:

Theorem 2.1. If modules of eigennumbers $\alpha_1, \ldots, \alpha_n$ of the linear part of mapping Φ which generated the endomorphism $T : \sum_n \to \sum_n$ are less than 1 and nonzero, nonresonansing, differently, then eigenvalues of T have the form $\lambda_{k=}\alpha_1^{k_1}\ldots\ \alpha_n^{k_n}$, where k=$(k_1, \ldots.k_n)$, $k_i \in Z_+$, $i = 1,\ \ldots\ ,\ n$, and corresponding eigensubspaces up to diffeomorphism are generated by the functions $f_k = z^k$ (consequently, all eigensubspaces are one dimensional).

3. Eigensubspaces of resonancing endomorphisms with resonancing monoms

We consider the algebra $\sum_2$ of series (formal or convergent series) of the form

$$\sum_{n,m} a_{n,m}x^n y^m\ ,$$

and endomorphism T of this space induced by formal series Φ, which module of eigenvalues of linear part of Φ are less than 1, i.e. we consider the operator of the form:

$$T : \sum_2 \to \sum_2, \quad f \to f\circ\Phi \quad (f \in \sum_2),$$

where eigenvalues $\propto_1$, $\propto_2$ of the linear part of Φ holds:

$$\propto_i : \qquad 0 < |\propto_i| < 1 \qquad (i = 1, 2)\,.$$

We will find eigenvalues of endomorphism T and we will calculate the dimensions of corresponding eigensubspaces. If λ eigenvalue for the endomorphism T, then by $E_T(\lambda)$ we denote the eigensubspace which corresponding to eigenvalue λ.

It is obviously resonancing conditions in this case have the form: either $\alpha_1 = \alpha_2^m$ $(m \geq 2)$, or else $\alpha_2 = \alpha_1^m$ $(m \geq 2)$. It is sufficiently consider the case $\alpha_1 = \alpha_2^m$, i.e.

$\alpha_1 = \alpha^m,\ \alpha_2 = \alpha,\ 0 < |\alpha| < 1,\ m \geq 2$. Since $m \geq 2$, then we have $\alpha_1 \neq \alpha_2$, and so linear part $\Phi_1$of the mapping Φ diagonalizable. Therefore, the linear part has the form $\Phi_1 = \begin{pmatrix} \alpha^m & 0 \\ 0 & \alpha \end{pmatrix}$, and according resconanceing monom has the form $w_2^m e_1 = \begin{pmatrix} w_2^m \\ 0 \end{pmatrix}$. Then according by Puancare – Dyulac theorem (see [1]) the mapping Φ may be transforming to the linear part Φ_1, and Φ will have the form $\Phi \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha^m x + cy^m \\ \alpha y \end{pmatrix}$, where either $c \neq 0$, or else $c = 0$ (when probably resconance monom there is not exist). In this section we consider the case when $c \neq 0$ (for otherwise case see next section), then the endomorphism will have the form $(Tf)(x,y) = f(\alpha^m x + cy^m,\ \alpha y)$. We consider the problem eigenvalues of the operator T, i.e. we will solve the equation: $f(\alpha^m x + cy^m,\ \alpha y) = \lambda f(x,y)$, where $f(x,y) = \sum_{k,l\geq 0} a_{k,l} x^k y^l$. So, we have:

$$(Tf)(x,y) = f(\alpha^m x + cy^m,\ \alpha y) = \sum_{k,l\geq 0} a_{k,l} (\alpha^m x + cy^m)^k (\alpha y)^l =$$

$$= \sum_{k,l\geq 0} a_{k,l} \alpha^l y^l \sum_{s=0}^{k} \begin{pmatrix} k \\ s \end{pmatrix} (\alpha^m x)^s (cy^m)^{k-s} = \sum_{l\geq 0} \sum_{k\geq 0} \sum_{s=0}^{k} a_{k,l} \alpha^{l+ms} \begin{pmatrix} k \\ s \end{pmatrix} c^{k-s} x^s y^{l+m(k-s)}.$$

Put $t = k - s \geq 0$; then $k = s + t$ and we have

$$Tf(x,y) = \sum_{l\geq 0} \sum_{s\geq 0} \sum_{t\geq 0} a_{s+t,l} c^t \alpha^{l+ms} \begin{pmatrix} s+t \\ s \end{pmatrix} x^s y^{l+mt} =$$

$$= \sum_{s\geq 0} x^s \left[\sum_{l\geq 0} \sum_{t\geq 0} a_{s+t,l} c^t \alpha^{l+ms} \begin{pmatrix} s+t \\ s \end{pmatrix} y^{l+mt} \right].$$

It is equal to

$$\lambda f(x,y) = \sum_{s\geq 0} x^s \left[\sum_{q\geq 0} \lambda a_{s,q} y^q \right].$$

So, for any $s \geq 0$ we have:

$$\sum_{l\geq 0} \sum_{t\geq 0} c^t a_{s+t,l} \alpha^{l+ms} \begin{pmatrix} s+t \\ s \end{pmatrix} y^{l+mt} = \left[\sum_{q\geq 0} \lambda a_{s,q} y^q \right].$$

If, we denote by $q = l + mt$, then $l = q - mt$ and left part of above equation has the form

$$\sum_{q\geq 0} \left[\sum_{\substack{l,t; \\ l+mt=q}} c^t a_{s+t,l} \alpha^{l+ms} \begin{pmatrix} s+t \\ s \end{pmatrix} \right] y^q.$$

Then for any $s \geq 0$ and $q \geq 0$ we have

$$\sum_{\substack{l,t;\\ l+mt=q}} c^t a_{s+t,l} \begin{pmatrix} s+t \\ s \end{pmatrix} \alpha^{l+ms} = \lambda a_{s,q}. \tag{3.1}$$

(3.1) is equivalent to:

$$(\alpha^{q+ms} - \lambda)\, a_{s,q} + \sum_{\substack{0 \leq l \leq q-1 \\ t \geq 1; \\ l+mt=q}} a_{s+t,l} c^t \begin{pmatrix} s+t \\ s \end{pmatrix} \alpha^{l+ms} = 0. \tag{3.2}$$

We will prove that $a_{s,q} = 0$ for any $s \geq 1$ and all $q \geq 0$.
Put $Q_0 = \{q \geq 0,\ \exists s \geq 0 :\ a_{s,q} \neq 0\}$ and $q_0 = min\{q : q \in Q_0\}$. If Q_0 is empty set, then all $a_{s,q} = 0$ and consequently $f = 0$. If Q_0 is nonempty set, then there exists s_0, such that $a_{s_0 q_0} \neq 0$ and for all $q < q_0$, $s \geq 0$ we have $a_{s,q} = 0$. Therefore, from (3.2) at $s = s_0$ and $q = q_0$ we have

$$(\alpha^{q_0+ms_0} - \lambda)\, a_{s_0,q_0} = 0,$$

and since $a_{s_0,q_0} \neq 0$, then we get $\lambda = \alpha^{ms_0+q_0}$. If we assume that $a_{s,q_0} \neq 0$ for some $\neq s_0$, so we have $\lambda = \alpha^{ms+q_0}$ and $\alpha^{ms+q_0} = \alpha^{ms_0+q_0}$. But it is impossible, since $0 < |\alpha| < 1$. In this case $q > q_0$ the equation (3.2) has the form;

$$\left(\alpha^{q+ms} - \alpha^{q_0+ms_0}\right) a_{s,q} + \sum_{\substack{0 \leq l \leq q-1 \\ t \geq 1; \\ l+mt=q}} a_{s+t,l} c^t \begin{pmatrix} s+t \\ s \end{pmatrix} \alpha^{l+ms} = 0. \tag{3.3}$$

We denote $Q_1 = \{q : q > q_0,\ \exists s \geq 0 :\ a_{s,q} \neq 0\}$ and $q_1 = min\{q : q \in Q_1\}$. If the set Q_1 is empty set, then all $a_{s,q}$ with $q > q_0$ are equal to zero. If, Q_1 is not empty set, then there exists s_1, such that $a_{s_1,q_1} \neq 0$ and $a_{s,q} = 0$ at $q_0 < q < q_1$ and for all s. Consequently, at the $q = q_1$, $s = s_1$ the equality (3.3) has the form:

$$(\alpha^{q_1+ms_1} - \alpha^{q_0+ms_0})\, a_{s_1,q_1} + \sum_{\substack{0 \leq l \leq q_1 \\ l+mt=q_1}} a_{s_1+t,l} c^t \begin{pmatrix} s_1+t \\ s_1 \end{pmatrix} \alpha^{l+ms_1} = 0. \tag{3.4}$$

If, $0 \leq l < q_0$, or $q_0 < \ l < q_1$, then we have $a_{s,l} = 0$, therefore $l = q_0$, and this case the sum on the left side of the equation (3.4) is equal to

$$\sum_{t:q_0+mt=q_1} a_{s_1+t,q_0} c^t \begin{pmatrix} s_1+t \\ s_1 \end{pmatrix} \alpha^{q_0+ms_1} = a_{s_1+\frac{q_1-q_0}{m},q_0} c^{\frac{q_1-q_0}{m}} \begin{pmatrix} s_1+\frac{q_1-q_0}{m} \\ s_1 \end{pmatrix} \alpha^{q_0+ms_1}.$$

There we have two cases:
Either $s_1 + \frac{q_1 - q_0}{m} = s_0$, then we have $ms_1 + q_1 = ms_0 + q_0$. Consequently, $\alpha^{q_1+ms_1} - \alpha^{q_o+ms_o} = 0$, so we have first addition in the (3.4) is equal to zero, and second addition is nonzero (since $a_{s_0,q_0} \neq 0$). But it is contradiction; or in the second case we have $s_1 + \frac{q_1 - q_0}{m} \neq s_0$, then we have $a_{s_1 + \frac{q_1 - q_0}{m}, q_0} = 0$, and consequently from (3.4) we get

$$\left(\alpha^{q_1+ms_1} - \alpha^{q_o+ms_o}\right) a_{s_1,q_1} = 0,$$

So, $\alpha^{q_1+ms_1} - \alpha^{q_o+ms_o} = 0$ and we have $q_1 + ms_1 = q_0 + ms_0$; it is contradiction to the condition:

$$s_1 + \frac{q_1 - q_0}{m} \neq s_0.$$

Finally, we have that only $a_{s_0,q_0} \neq 0$.
That means, if for some pair (s_0, q_0) holds $a_{s_0,q_0} \neq 0$, then $\lambda = \alpha^{ms_0+q_0}$ and $a_{s,q} = 0$ for all pairs $(s, q) \neq (s_0, q_0)$. Consequently, eigenvalues of the endomorphism T have the form $\lambda = \alpha^{q_0+ms_0}$, and eigenfunctions have the forms $f_{(s_0,q_0)} = x^{s_0} y^{q_0}$, i.e.,

$$Tf_{(s_0,q_0)}(x, y) = \lambda f_{(s_0,q_0)}(x, y) = \alpha^{ms_0+q_0} x^{s_0} y^{q_0}.$$

However:

$$Tf_{(s_0,q_0)}(x, y) = (\alpha^m x + cy^m)^{s_0} (\alpha y)^{q_0} = \sum_{k=0}^{s_0} \binom{s_0}{k} (\alpha^m x)^k (cy^m)^{s_0-k} \alpha^{q_0} y^{q_0} =$$

$$= \sum_{k=0}^{s_0} \binom{s_0}{k} c^{s_0-k} \alpha^{q_0+mk} x^k y^{q_0+ms_0-mk}.$$

It must be equal to $\alpha^{q_0+ms_0} x^{s_0} y^{q_o}$, so we have $\sum_{k=0}^{s_0} \binom{s_0}{k} c^{s_0-k} \alpha^{mk} x^k y^{m(s_0-k)} \equiv \alpha^{ms_0} x^{s_0}$ (after cancellation to $x^{q_0} y^{q_0}$) . Since the right side independent of y, then we conclude that the left side, also, independent of y ; consequently, for any $k < s_0$ we have:

$$\binom{s_0}{k} c^{s_0-k} \alpha^{mk} x^k = 0;$$

Since $c \neq 0$ and $\alpha \neq 0$, it is possible only, if $s_0 = 0$.
That means, in this case eigenvalues of endomorphism T have the form $\lambda_q = \alpha^q$, $q \geq 0$, and eigenfunctions have the forms $f_q(x, y) = y^q$. There are not another eigenvalues and eigenfunctions. Thus $dim E_T(\alpha^q) = 1$.

Remark 3.1. Analogously, if we have otherwise resonancing condition , i.e., $\alpha_2 = \alpha_1^m$ ($m \geq 2$), then it is clear that the eigenfunctions, which corresponding to eigenvalues $\lambda_q = \alpha^q$, $q \geq 0$, have the forms $f_q(x, y) = x^q$.
Consequently, in the resonancing case with the resonancing monoms we proved next theorem:

Theorem 3.1. In the resonancing cases with the resonancing monoms every eigenvalue of endomorphism $T : \sum_2 \to \sum_2$ has the form $\lambda_q = \alpha^q$ (where q is nonnegative whole number) and corresponding eigenfunction has the form $f_q(x, y) = y^q$ (or has the form $f_q(x, y) = x^q$). Consequently, corresponding eigensubspaces are one-dimensional.

4. Eigensubspaces of resonancing endomorphisms without resonancing monoms

Now we consider the case $c = 0$, i.e., the case, when resonancing endomorphisms has not any resonancing monoms. Then, again according by Puancare – Dyulac theorem mapping Φ reduced by diffeomorphic transformation to the normal form $\Phi = \begin{pmatrix} \alpha^m & 0 \\ 0 & \alpha \end{pmatrix}$, where $m \geq 2$. Then we have:

$$(Tf)(x,y) = f(\alpha^m x, \alpha y) = \sum_{k,l \geq 0} a_{k,l} \alpha^{mk} x^k \alpha^l y^l = \sum_{k,l \geq 0} a_{k,l} \alpha^{mk+l} x^k y^l,$$

and it must be equal to

$$\lambda f(x,y) = \lambda \sum_{k,l \geq 0} a_{k,l} x^k y^l.$$

Consequently, for any $k, l \geq 0$ we have:

$$(\alpha^{mk+l} - \lambda) a_{k,l} = 0.$$

If, for some coefficient a_{k_0,l_0} holds $a_{k_0,l_0} \neq 0$, then we have $\lambda = \alpha^{mk_0+l_0}$; if , there is another coefficient a_{k_1,l_1}, such that is not zero too, then again holds $\lambda = \alpha^{mk_1+l_1}$, so we have $\alpha^{mk_0+l_0} = \alpha^{mk_1+l_1}$ and faithfully $mk_0 + l_0 = mk_1 + l_1$, i.e., $(l - l_0) = m(k_0 - k_1)$.

Let $l_0 = \min\{l : l > 0, \exists k : a_{k,l} \neq 0\}$. Then there exists k_0 , such that $a_{k_0,l_0} \neq 0$. If, there exists another coefficient $a_{k,l}$, such that $a_{k,l} \neq 0$ for any pair $(k,l) \neq (k_0,l_0)$, then we have $(l - l_0) = m(k - k_0) > 0$; so for pairs (k,l) are satisfied the conditions: $k < k_0$ and $l = l_0 + m(k_0 - k)$. Then the eigenfunction $f(x,y)$ has the form:

$$f(x,y) = \sum_{k=0}^{k_0} a_{k,l_0+m(k_0-k)} x^k y^{l_0+m(k_0-k)}.$$

Therefore, we have subspace consists of eigenfunctions with respect to eigenvalue $\lambda = \alpha^{mk_0+l_0}$, which dimension is equal to $(k_0 + 1)$.
Thus, in this case we conclude that, the eigenvalues of endomorphism T again has the form $\lambda_q = \alpha^q$, $q \geq 0$. But, corresponding subspace $E_q = E_T(\lambda)$ consists of eigenfunctions induces by monoms $x^k y^l$, where $k, l \geq 0$ and $mk + l = q$. Let $q = mk_0 + l_0$, where $0 \leq l_0 \leq m$. Then we have $k_0 = \left[\frac{q}{m}\right]$ (where, $[a]$ defined the integer part of the a).
Therefore we conclude the $dimE_q$ is equal to $\left[\frac{q}{m}\right] + 1$ and eigensubspace E_q consists of polynoms, which have the forms:

$$f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} x^k y^{q-mk}.$$

Remark 4.1. Analogously, if we have otherwise resonancing condition , i.e., $\alpha_2 = \alpha_1^m$ $(m \geq 2)$, then it is clear that the eigenfunctions, which corresponding to eigenvalues $\lambda_q = \alpha^q$, $q \geq 0$, have the forms $f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} y^k x^{q-mk}$.

Consequently, in the resonancing case without resonancing monoms we proved next theorem:

Theorem 4.1. In the resonancing cases without resonancing monoms every eigenvalue of endomorphism $T:\sum_2 \to \sum_2$ has the form $\lambda_q = \alpha^q$ (where q is nonnegative whole number) and corresponding eigenfunctions havre the forms $f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} x^k y^{q-mk}$ (or has the form $f(x,y) = \sum_{k=0}^{\left[\frac{q}{m}\right]} a_{k,q-mk} y^k x^{q-mk}$). Consequently, corresponding eigensubspaces are ($\left[\frac{q}{m}\right]+1$)- dimensional, where m is an order of resonancing conditions.

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