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Fractal Conception Evaluation of Blood-Vessel System State in the Anterior Part of an Eye

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Abstract. The paper deals with parametrization of graphical representation in the anterior part of an eye, analysis of the systems that perform statistical analysis, information processing, diagnostics and data bank creation. Information about fractal conception, the effect of its application and complex of necessary theoretical and practical works in this field are given. The use of the fractal size of the eye vessels as an information wrapping method and the perspective of linear regression or the least square methods are studied. The efficiency of the use of fractal concept for the preservation and processing of graphical representation of the blood-vessel system of the anterior part of an eye is shown.

Key Words and Phrases: blood-vessel system of an eye, image recognition, simulation of chaotic structures, fractal structures, simulation.

2010 Mathematics Subject Classifications: 34L10, 41A58, 46A35

1. Introduction

Numerous historical facts from ancient medicine can be considered as important facts of eye-diagnostics information. There is also scientific evidence that there is a correlation between the visual analysis of changes in the anterior part of an eye and internal diseases and even some psychological disorder symptoms.

Today, there is a great believance that the functional changes and hormonal shortcomings that occur in interior organs and other objective factors are revealed. The importance given to the perspective of the eye that the blood vessel structure or changes in it can be instrumental in early prediction of internal diseases is developing on a day-to-day. One of the reasons of this tendency is related to the achievements of electronic appliances and medical devices in medicine, and the other reason is the high achievement of modern information technologies and the broad range of mathematical-cybernetic methods.

As the symptomatic factors as the appearance of yellow strains in the eye is usually more common in liver pathology, swelling in the eyes in the blood-vessel cardiological diseases, redness in the eyes in hypertension have been known for a long time, now there are more complex requirements based on external showings of the eye and they consider

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to study pathalogical changes more thouroughly. Local vein blockage in the eye is one of the factors that can dramatically alter the appearance of the anterior part of the eye. Today the intuitive conviction based on subjective observation of the people as "Eyes are the light of life", "The eyes are the mirror of the heart" needs to be transferred to a more serious and objective evaluation. Analysis of the existing scientific literature in this field shows that analysis of an eye-vascular system based on mathematical-cybernetic methods can be considered as a perspective direction that creates important steps in the field of medical diagnostics [2].

2. The most important apparatus for the automatic examination of the eye's blood-vascular system image

A positive solution to the problem of objective parametrization (characterization) of the eye's blood-vessel system can refer to the medical examination tools available in the current eye care. A positive solution to the problem of symptomatic diagnostic problem is undoubtedly dependent on the level of improvement of the analyzers and measurement systems currently used in the eye examination and treatment. The analysis of survey reviewer of technical literature, which we have undertaken in this field, does not cover the issue, but it can to a certain extent give the most important idea of the general situation.

Some of the apparatus manufactured by leading companies of advanced countries in the field of medical devices can play a major role in the diagnostics, prophylaxis and treatment prophylaxis and treatment of eye diseases and enable to carry out fundamental researches in the field of examination of blood-vascular system of the eye. Many of these equipments were designed to examine the anterior part the eye, the ultrasound examination of the eyeground, the visual field and the visual acuity. The last model apparatus manufactured in Japan, Korea, the USA, Central Europe and other countries are widely used in modern medicine. The Japanese "TOPCON" company's test, measurement, treatment complex is successfully used in the diagnostics, prevention and treatment of eye diseases. The "FOROPTER" complex is a computerized intellectual system, has the function of evaluating the patien's vision area and sharpness and is widely used in clinical practice. Germanc "VOLK" and "Reister" systems also examine the eyeground, "ALKON", "LAUREAT" systems are able to carry out the most recent cataract surgery. The "Quantel" system of France is used in the prevention and treatment of degenerative diseases of retina.

Of course it is impossible to claim that mathematical-cybernetic methods such as statistical analysis, image recognition and mathematical simulation are used as a fundamental necessity in addressing the broad spectrum of mentioned systems. On the other hand, there is no need to note that the functional issues such as simulation with differential functions, arising in operational control in the dynamics of processes during the patient's examination, and the analysis of transition state in the patient are widely used. For example, the German-made "Zeiss" brand equipment examines the activity of the blood stream in retinal vessels. For this examination, the patient is administrated intravenously 2ml-25 % or 5ml-10 % fluorescein sodium salt. Fluorescein enters ocular circulation from the internal carotid artery through the eye artery. Fluoresein first enters the choroidal vessels,

Figure 1: A picture of the unique structure of the eye's blood-vessel system received from optic devices

then to arteries and veins of retina. The drug is injected within 6 seconds and appears in fluorescein vision nerve and choroid within 8 to 11 seconds. The duration of fluorescein entering the veins depends on the age of the patient, the patient's cardiovascular status, and the rate of fluorescein entering. White and black images are taken within 10 seconds after injection, 1 image in every second during 20 seconds. Then the patient is offered 10-minute rest. Then 5-10 images are taken. In some cases, the pictures taken in 15 minutes can also be used as useful diagnostic information [9].

Thus, it can be shown from this example that during the examination, both the recording of the reactions arising from the dynamic effects in the body and the effects of intermediate stages of examination require to have and storage the graphical description by parametrizing it. With regard to the problem of blood vessel examination of the eye, it should be noted that for both diagnostic and statistical analysis, reduction of optic at and ultrasound examination results as physical measure information to a compact form, in other words, parametrization does not manifests itself to day.

3. Possible folding methods for the eye's blood-vasculus system image. Fractal reflection

The picture shows a blood-vessel-eye system's photo surrounded by different size square networks. The distinctive thickness of vessel and formation of their dendrites with random character forms draws attention.

If one or more dendrite aggregates are taken within each check, each of them can be regarded as an implementation of a random function, and we can suggest that we can centralize multiple implementations on square networks. If this implementation is carried out on a basis of any methods, we can obtain folding of the image either in the form of any numerical characteristic of characteristic function. Such statement of the problem first of all focuses on the Fourier spectral analysis that requires complex, graphical processing processes such as separately analyzing the image of the dendrite aggregate falling on each network, subsequent centering and defecting the spectral composition. It should be noted that this issue itself will create an important stage and will require independent algorithmic works to find its solution.

The another direction may be calculation of general ℓlength of vessels with respect to δin arbitrary $\delta + d\delta$ interval within the network of the given size. The Fourier transform of this distribution function gives a complex variable characteristic function, and in principle, such a substitution can be used as an image folding method.

It should be noted that as a mathematical description of dendritic structures, there is a scientific study showing that fractal notation of such structures is far more effective [7,8]. In this study, the total volume of chaotic pores channel, surface area and effective perimetry concepts were introduced and they were used to parametrize the dendritric channel system. Fig. 2.

Fractal is a fraction - dimensional object, i.e. a whole consisting of its own parts Benua Mandelbrot proposed initial guidelines for geometrical measurement or calculation of such structures [5]. These rules and formulas are now also available.

According to Mandelbrot, the fractal is a structure recurring in scale changes from the biggest to the smallest and where any geomterical object is considered. When ordinary geomterical objects (full size) are broken down into similar parts the resulting size for example total length, total surface area or total volume remain unchanged. In fractal objects (fractional dimensional) this dimension varies with the fact that the fractal dimension differs from the topological dimension. In the quantitative sense fractal property is reflected just in the mentioned difference. It should be noted that the size of the fractality may be both greater or less than the topological dimension.

Divide sequentially the identified sides of any D- dimensional geometrical structure into Mequal parts. After divisions as each iteration the original (or any element obtained from dividend) turns into N number identical element, we can write:

$$N = M^D \tag{1}$$

It is obvious that in objects with no fractality, D equals 1, 2 or 3. Now in addition to some variants of fractality that have become classic examples, the expanding diversity is successfully applied not only to non-traditional computer graphics, but to modeling of non-traditional objects, too.

The most common formula for determining the fractal dimension can be written using formula

$$D = \frac{\ln M}{\ln M} \tag{2}$$

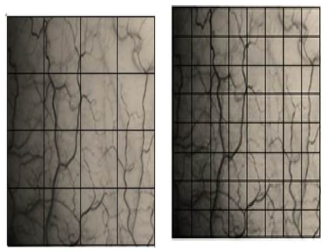
Here D is a fractal dimension [5,6].

At present, there are numerous algorithms based on different approaches to determine the dimension of fractals in nature [3,7,8].

It is clear from Fig. 1 that vascular system in the eye forms one natural fractal system. Here, the main problem is the determination of the regular fractal structure of the system equivalent to it in this or other point of view and the solution of parametrization problem on this basis. In our study, we aimed to determine the fractal dimension of blood vessels in the eye and the length of blood vessels to parametrize the eye. At first, in order to determine the length of blood vessels, we measure by taking two points of the vessel and determine the length along any straight line. But, the result obtained at this time may be considered as an approximate result. If the distance between the points is taken smaller, the size will be adjusted. We can define the total length of blood vessels in the eye based on the formula used by Lewis Fry Richardson in measuring the geographical borders of the countries and the algorithm suggested below. We can also define the fractal dimension according to the length of blood vessels in the eye, i.e. by the length we can determine the fractal dimension. Lewis Fry Richardson has shown that the L length of the borders of the countries varies depending on the δ scale of the map, $L = L(\delta)$. We can use this formula for the blood vessel system we consider [8].

$$L(\delta) = A\delta^{1-D}. \tag{3}$$

Here A is a constant, D is a fractal dimension.



a) b)

Figure 2: Statistical processing of eye-vascular system image based on reflection on different dimensional networks

E. Feder has shown on the basis of different experiments that the following formula can be used to determine any δ –dependent length [6]:

$$m_i\delta_i = A\delta_i^{1-D}. \tag{4}$$

Here $L(\delta) = m\delta$, where m sizes for different scale δ were obtained. Since we do not have a scale in the graphical description of the considered blood vascular system, if we place on the graphic representation of a square grid with the side of length δ_1 (fig 2. a)), then a quadratic grid with side of length δ_2 (fig. 2. b)), and in this sequence a quadratic grid with side of length δ_n , then for each δ_1 δ_2.....δ_n we obtain the length of vessels, m_1 m_2 .....m_n. It is appropriate to take $n \geq 5$. Here m_i is the sum length of the parts of vessels that the quadratic grid cuts. Here each δ value has a specific $m\delta$ length, and if we draw a graph of these values, we can see their linear dependence. For each $\lg(\delta)$ we get certain $\lg(m\delta)$. If based on

$$x = \lg\delta; y = \lg(m\delta), \tag{5}$$

we draw a graph, we can see linear dependence in the form of $y = ax + b$ [8]. So, by the least square method we can find the constants a and b

$$a = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{n\sum x_i^2 - \sum x_i \sum x_i};$$

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \sum x_i \sum x_i}.$$

From the known $L(\delta) = m\delta$ and formula (5) we get $y = \lg(m\delta) = \lg L(\delta)$. Then we can write $y = ax + b$ in the form $\lg L = a\lg\delta + b$. Hence we get

$$10^{\lg L} = 10^{a\lg\delta + b};\; L = 10^b\delta^a. \tag{6}$$

Taking (3) into account, we find $A = 10^b$; $a = 1-D$. From the expression $D = 1-a$ we can determine the fractal dimension with respect to the length of the blood vessel according to the graphic description of the anterior of the eye. Using the quantities D and A being the found fractal dimension in length by means of the formula $L(\delta) = A\delta^{1-D}$ (3), we can determine L length of the vessels according to the graphic description of the anterior part of the eye for any $\delta \times \delta$- dimensional square grid.

The carried out investigations enable to determine two unique values, the length and fractal dimension of the vascular system of the anterior part of the eye. Thus, by the unique value corresponding to each graphic description of the anterior part of the eye, we can parametrize this description.

4. Conclusion

Modern development of information technology shows that parametrization of graphic descriptions in automation of medical diagnostics is more likely to provide possible information. The studies have shown that as the analysis of graphic description complicates, the focus is on the parametrization of description in studying diagnostic descriptive bank, statistical analysis and the dynamics of the description. Today, fractal description analysis as a separate field of science involves professionals in the field of information technology and is used in many fields including graphic descriptions. Parametrization of the graphic description of the anterior part of the eye using the fractal analysis apparatus may be used to examine the anterior part of the eye and to study the dynamics of pathological conditions. The achievements in the field of finding these parameters with certain accuracy can eventually be an important step in ensuring a positive solution to the problem of computerized diagnostics.

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