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Examples of the Discrete Additive Derivative of the Second-order Discrete Multiplicative Derivative

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Abstract. As is known, in a discrete analysis there are investigated either discrete processes or continuous processes with discrete analog. In a discrete process various properties of sequences are studied. They can be arithmetic, geometric progressions and the Fibonacci sequence. Determination of the general term of an arithmetic progression leads us to the Cauchy problem for first-order equation with discrete additive derivative; determining the general term of a geometric progression brings us again to the Cauchy problem for a first-order equation with discrete multiplicative derivatives. Finally, the definition of the general term leads us to the Fibonacci sequence for the Cauchy problem for second-order equations with discrete additive derivatives.The main objective of the discrete analysis is the discretization of mathematical models derived from the continuous analysis and study of the resulting discrete model. Multiplicative derivative and integrated, integral, compact and simple properties are given in three or four pages in [8]. Thus, the multiplicity properties were expected here rather than additivity. It is shown that "derivation of derivatives, derivatives" and "production of integers are integral". The distinctive (marking) of the integral belongs to us.

Key Words and Phrases: discrete additive analysis, discrete multiplicative analysis, additivo-multiplicative and multiplicative-additive equations.
2010 Mathematics Subject Classifications: 35J25, 35B45,42B20, 47B38

1. Introduction

The derivative taught in "Algebra and the beginning of analysis" in secondary school and "Mathematical Analysis" course in Higher School is mainly additive derivative [1, 2]. Although the multiplicative derivative has been created for around nearly a century [3], problems for the multiplicative derivative equations have been considered recently [4, 5]. Here we will talk about the discrete cases of these additive and multiplicative derivatives [6, 7, 8]. We began to look at the problems for ordinary discrete additivo-multiplicative and multiplicativo - additive derivative equations, [9, 10, 11]. It should be noted that the markings for discrete derivatives and integrals also belong to us [12].

Here we look at Cauchy and boundary value problems for a two-dimensional third-order equation (the second-order discrete additive relative to one variable, which holds a discrete multiplicative derivative relative to the other variable).

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2. Solution of the problem

Such third-order equations as

$$\left(y_n^{('')}\right)^{[']} = f_n, n \geq 0, \tag{1}$$

are considered. Here also f_n $n \geq 0$ is the given sequence and y_n $n \geq 0$ is the search sequence. Using the definitions of derivatives, we'll get

$$y_{n+3} = f_n \cdot \frac{y_{n+2}^2}{y_{n+1}} + y_n \frac{y_{n+2}^3}{y_{n+1}^3}, n \geq 0, \tag{2}$$

Here, emphasizing "n", it becomes possible to determine all y_ns beginning from y_3 with the help of y_0, y_1 and y_2 (dependence from the f_n is also available). But it is impossible to give the analytical note for the general solving of (2).

3. The solution is solved by integration

That is why, returning to the (1) and using the discrete additive derivative:

$$y_{n+1}^{[']} - y_n^{[']} = f_n, n \geq 0, \tag{3}$$

Here, dropping out "n":

$$y_1^{['']} - y_0^{['']} = f_0,$$
$$y_2^{['']} - y_1^{['']} = f_1,$$
$$\vdots$$
$$y_n^{['']} - y_{n-1}^{['']} = f_{n-1},$$

Adding these expressions, we'll get :

$$y_n^{['']} - y_0^{['']} = \sum_{k=0}^{n-1} f_k,$$

or

$$y_n^{['']} = y_0^{['']} + \sum_{k=0}^{n-1} f_k, n \geq 1, \tag{4}$$

Here, using the designation

$$g_n = g_n\left(y_0^{['']}, f_k\right) = y_0^{['']} + \sum_{k=0}^{n-1} f_k, n \geq 1, \tag{5}$$

the equation will change to

$$y_n^{[']} = g_n,\ n \geq 1, \tag{6}$$

So, the given three-order equation (1)is brought to the two-order equation (6). Using the definition of the discrete multiplicative derivative in the last equation, we'll get:

$$\frac{y_{n+1}^{(\prime)}}{y_n^{(I)}} = g_n, \quad n \geq 1, \tag{7}$$

changing "n":

$$\frac{y_2^{[\prime]}}{y_1^{[\prime]}} = g_1,$$

$$\frac{y_3^{[\prime]}}{y_2^{[\prime]}} = g_2,$$

$$\vdots$$

$$\frac{y_{n-1}^{[\prime]}}{y_{n-2}^{[\prime]}} = g_{n-2},$$

$$\frac{y_n^{[\prime]}}{y_{n-1}^{[\prime]}} = g_{n-1},$$

Multiplying these expressions, we get

$$\frac{y_n^{[\prime]}}{y_1^{[\prime]}} = \prod_{s=1}^{n-1} g_s,$$

or

$$y_n^{[\prime]} = y_1^{[\prime]} \prod_{s=1}^{n-1} g_s, n \geq 2, \tag{8}$$

Here, just like in (3), using the designation

$$h_n = h_n\left(y_1^{[\prime]} g_s\right) = y_1^{[\prime]} \prod_{s=1}^{n-1} g_s, n \geq 2, \tag{9}$$

the equation (7) will change to

$$y_n^{[\prime]} = h_n, n \geq 2, \tag{10}$$

Applying the discrete additive derivative on this equation once more, we'll get

$$\frac{y_{n+1}}{y_n} = h_n, n \geq 2,$$

changing "n":

$$\frac{y_3}{y_2} = h_2,$$

$$\frac{y_4}{y_3} = h_3,$$

$$\vdots$$

$$\frac{y_{n-1}}{y_{n-2}} = h_{n-2},$$

$$\frac{y_n}{y_{n-1}} = h_{n-1},$$

Multiplying them, we get

$$\frac{y_n}{y_2} = \prod_{m=2}^{n-1} h_m,$$

or

$$y_n = y_2 \cdot \prod_{m=2}^{n-1} h_m, \quad n \geq 3, \tag{11}$$

So, we achieve:

Theorem 1. *If* f_n $n \geq 0$ *is the given valid elemental sequence, the equation* (1) *will have its solving and it is like* (10), *so* $h_n s$ *are like in* (9) *and* $g_n s$ *are like in* (4), y_0'', y_1^I *and* y_2 *are optional constants.*

4. Cauchy problem

If the initial conditions

$$y_k = \alpha_k, k = \overline{0,2}, \tag{12}$$

are added to the given third-order equation (1), then, because of

$$y_0^{[\prime\prime]} = \frac{y_0 y_2}{y_1^2} = \frac{\alpha_0 \alpha_2}{\alpha_1^2}, y_1^{[\prime]} = \frac{y_2}{y_1} = \frac{\alpha_2}{\alpha_1}, \tag{13}$$

(1), (10) solving of the Koshi example is defined from (10) as

$$y_n = \alpha_2 \cdot \prod_{m=2}^{n-1} h_m, \quad n \geq 3, \tag{14}$$

So, h_n-s and g_s-s are defined from (4) and (8), taking into consideration (12).

Theorem 2. *Under the terms of Theorem 1, if* α_k, $k = \overline{0,2}$, *the Koshi example has the only solving and this is given with the help of* (13), *so* $h_n s$ *are defined with the help of* (8) *and* g_s*-s – with the help of* (8) *in the* (4).

5. Boundary problem

Now, taking the $\overline{0, N-3}$ numbers of n in (1), let's see the border conditions of the equation:

$$y_0'' = \alpha, y_1' = \beta, y_N = \gamma, \tag{15}$$

Taking into consideration (14) in (4) and (8):

$$g_n = \alpha + \sum_{k=0}^{n-1} f_k, n \geq 1, \tag{16}$$

$$h_n = \beta \cdot \prod_{s=1}^{n-1} g_s, n \geq 2, \tag{17}$$

Designations of (15) and (16) define g_ns and h_n-s as equal, that is, there's no discretion.

Finally, taking into consideration the general solving of (10) in the third of (14) border conditions, we get

$$\gamma = y_N = y_2 \prod_{m=2}^{N-1} h_m,$$

and

$$y_2 = \frac{\gamma}{\prod_{m=2}^{N-1} h_m}, \tag{18}$$

The general solving of the border example is possible from (**??**) general solving with the help of (16)

$$y_n = \frac{\gamma}{\prod_{m=2}^{N-1} h_m} \cdot \prod_{m=2}^{n-1} h_m = \frac{\gamma}{\prod_{m=n}^{N-1} h_m}, \tag{19}$$

So, we get:

Theorem 3. *Under the terms of Theorem 1, if the given α, β and γ are the true given numbers, there's the only solving of the border example* (1) *and* (14), *and this solving is like* (18). *So, $h_n s$ are given with the help of* (16) *and $g_n s$ – with the help of* (15).

6. Results

Here, the third order presents the Cauchy and boundary problems for the equation with discrete nonlinear differences, and the analytical expressions for the solution of these problems are obtained.

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