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Investigation of Propagation of Nonlinear Waves in a Structure Consisting of Cylindrical Net System

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Abstract. Cylindrical net movement at the smooth cylinder have been obtained on the base of the general net motion theory. On the next basis system of the vectors: in direct of cylinder axes; in tangential (rotated) to a cross-section of the cylinder: perpendicular (rotated) to the cylinder axes. The case of the relative symmetrical filaments position is taken. In this research work the strain impact to the net is considered. The task comes to the hyperbolic system of equations at corresponding conditions. As far as the larger significance of parameters corresponds larger speed of wave spreading, that leads to jumping on the front. To solve the task at the front, there are using the law of mass preservation and law of a motion quantity changing to find out the jump spread speed as a function of incline of filament from cylinder axis and speed of the impact.

Key Words and Phrases: wave front, spread speed, law of a motion, cylindrical base, net motion, tension, angular acceleration

1. Introduction

On the basis of Kh.A. Rakhmatulin's equations on the motion of a filament, the equations of motion of the net were obtained [1, 2]. On the dynamics of the netthere are solved a number of flat and spatial problems in a rectangular Cartesian coordinate system [3, 4, 5, 6, 7]. Here we consider the problem of the motion of a net on a cylindrical base. In addition to the theoretical interest, the problem is of practical importance, for example, the dynamics of flexible drill pipes.

2. General equations of net motion

The equation of motion of the net taking into account the reaction of the supporting body and the geometric relationships will have the form unlike [2].

$$\frac{\partial}{\partial S_1}\left(\sigma_1\overline{\tau_1}\right)+\frac{\partial}{\partial S_2}\left(\sigma_2\overline{\tau_2}\right)=\rho\frac{\partial^2\overline{r}}{\partial t^2}+p\overline{n}$$

$$\left(1+e_1\right)\overline{\tau_1}=\frac{\partial\overline{r}}{\partial S_1};\qquad \left(1+e_2\right)\overline{\tau_2}=\frac{\partial\overline{r}}{\partial S_2}. \tag{1}$$

Here $\overline{r}$ – is the radius of the particle of the net particle, P is the reaction force of the cylinder l_1, l_2 – are the relative elongations corresponding to the properties of the filaments, S_1, S_2 – Lagrangian coordinates of the particles of filaments, σ_1, σ_2 – are conditional stresses, defined as the sum of the tension of individual threads of one family (intersecting a section of a filament of another family), referred to the original length of the element in question.

Such a distribution of mass and effort is permissible with a sufficiently dense net, ρ – is the mass of the net per unit area in the initial state, $\overline{\tau}_1$,$\overline{\tau}_2$ - are the unit vectors tangent to the filaments, $\overline{n}$ - is the normal to the surface of the cylindrical base.

3. Coordinate system

A basis of a cylindrical system is taken: a unit vector $\overline{i}$ - parallel to the axis of the cylinder, $\overline{j}$ – the unit vector of the tangent(rotating) to the cross section of the cylinder, $\overline{k}$ – unit vector perpendicular(rotating) to the previous ones.

Then

$$\overline{\tau_1} = \cos\gamma_i\overline{i} \ + sin\gamma_1\overline{j};\ \overline{\tau_2} = cos\gamma_2\overline{i} + \sin\gamma_2\overline{j}, \tag{2}$$

where $\gamma_{1,2}$- the filament angles formed with the axis of the cylinder.

Derivatives

$$\frac{\partial\overline{\tau_1}}{\partial S_1} = \cos\gamma_i\frac{\partial\overline{i}}{\partial S_1} + \overline{i}\frac{\partial(cos\gamma_1)}{\partial S_1} + sin\gamma_i\frac{\partial\overline{j}}{\partial S_1} + \overline{j}\frac{\partial(sin\gamma_1)}{\partial S_1}$$

$$\frac{\partial\overline{\tau_2}}{\partial S_2} = cos\gamma_2\frac{\partial\overline{i}}{\partial S_2} + \overline{i}\frac{\partial(cos\gamma_2)}{\partial S_1} + sin\gamma_2\frac{\partial\overline{j}}{\partial S_2} + \overline{j}\frac{\partial(sin\gamma_2)}{\partial S_2}$$

Or considering

$$\frac{\partial\overline{i}}{\partial S_1} = \frac{\partial\overline{i}}{\partial S_2} = 0;\ \frac{\partial\overline{j}}{\partial S_1} = \frac{sin\gamma_1}{r}\overline{k}\,\frac{\partial\overline{j}}{\partial S_2} = -\frac{sin\gamma_2}{r}\overline{k}$$

We get

$$\begin{aligned}\frac{\partial\overline{\tau_1}}{\partial S_1} &= \frac{\partial(cos\gamma_1)}{\partial S_1}\overline{i} + \frac{sin\gamma_1^2}{r}\overline{k} + \frac{\partial(sin\gamma_1)}{\partial S_1}\overline{j}\\ \frac{\partial\overline{\tau_2}}{\partial S_2} &= \frac{\partial(cos\gamma_2)}{\partial S_2}\overline{i} - \frac{sin\gamma_2^2}{r}\overline{k} + \frac{\partial(sin\gamma_2)}{\partial S_2}\overline{j}\end{aligned} \tag{3}$$

Also taking into account $\overline{r} = x\overline{i} + r\overline{k}$, we have

$$\frac{\partial\overline{r}}{\partial t} = \frac{\partial x}{\partial t}\overline{i} + r\omega\overline{j}$$

$$\frac{\partial^2\overline{r}}{\partial t^2} = \frac{\partial^2 x}{\partial t^2}\overline{i} + r\varepsilon\overline{j} + r\omega^2\overline{k} \tag{4}$$

ω – angular velocity, ε – angular acceleration

4. Equations of motion of a cylindrical net

Substituting (3) and (4) into (1) we obtain

$$\frac{\partial}{\partial S_1}\left(\sigma_1 \cos\gamma_1\right)+\frac{\partial}{\partial S_2}\left(\sigma_2 \cos\gamma_2\right)=\rho\frac{\partial^2 x}{\partial t^2} \tag{5}$$

$$\frac{\partial}{\partial S_1}\left(\sigma_1 sin\gamma_1\right)+\frac{\partial}{\partial S_2}\left(\sigma_2 sin\gamma_2\right)=r\varepsilon$$

$$\frac{\sigma_1}{r}sin\gamma_1^2-\frac{\sigma_2}{r}sin\gamma_2^2=p+\rho r\omega^2$$

Next, the symmetrical arrangement of the right and left fibers is considered. Then equations (5), taking $\sigma_1=\sigma_2=\sigma,\quad \gamma_1=-\gamma_2=\gamma,\quad \omega=0,\ \ \varepsilon=0,$
will take the form

$$2\frac{\partial}{\partial S}\left(\sigma,\cos\gamma\right)=\rho\frac{\partial^2 x}{\partial t^2} \tag{6}$$

$$2\sigma sin\gamma=p$$

5. Geometric relations

We define the derivative of the radius vector $\bar{r}$ in S. Denoting $\bar{r}=x\bar{i}+r\overline{k},$

$$\frac{\partial\bar{r}}{\partial S}=\frac{\partial x}{\partial S}\bar{i}+r\frac{\partial\overline{k}}{\partial S}=\frac{\partial x}{\partial S}\bar{i}+\frac{\partial y}{\partial S}\bar{j}$$

y –circular coordinate, where according to (1) and (3)

$$\frac{\partial x}{\partial S}=\left(1+e\right)cos\gamma \tag{7}$$

$$\frac{\partial y}{\partial S}=\left(1+e\right)sin\gamma \tag{8}$$

Since the net does not rotate, then $y=const$, and

$$\frac{\partial\left[\left(1+e\right)\ sin\gamma\right]}{\partial t}=0$$

or

$$\left(1+e\right)sin\gamma=sin\gamma_0 \tag{9}$$

6. Stretching blow on the cylindrical net

Let the infinite unloaded net (Fig. 1) be driven from one end with a constant velocity v.

Since waves with greater deformation propagate faster than waves with less deformation, the wave front will undergo a jump (8). Assuming that the motion is self-similar, we have.

$$\xi = \frac{S}{bt}; \quad x = btf(\xi);$$

$$\frac{\partial \xi}{\partial t} = -\frac{S}{bt^2}; \quad \frac{\partial x}{\partial t} = bf(\xi) - btf'(\xi)\frac{S}{bt^2} = bf(\xi) - \frac{S}{t}f'(\xi)$$

$$\frac{\partial^2 x}{\partial t^2} = \frac{b}{t}\xi^2 f''(\xi) \tag{10}$$

Substituting (10) into (5), we obtain

$$2(\sigma cos\gamma)' = \rho\xi^2 f'' \tag{11}$$

$$(1+e)\, cos\gamma = f' \tag{12}$$

Substituting (9) into (2) with $\sigma = Ee$, we get $sin\gamma_0 ctg\gamma = f'$;

$$\left[cos\gamma\left(\frac{sin\gamma_0}{sin\gamma} - 1\right)\right]' E = \rho\xi^2 f''. \tag{13}$$

In (13), eliminating f, obtaining

$$\left[cos\gamma\left(\frac{sin\gamma_0}{sin\gamma} - 1\right)\right]' E = \rho\xi^2 sin\gamma_0 ctg'\gamma$$

or

$$sin\gamma_0 ctg'\gamma - cos'\gamma = \frac{\xi^2}{a^2} sin\gamma_0 ctg'\gamma$$

or

$$-sin\gamma_0 csc^2\gamma \bullet \gamma' + sin\gamma \bullet \gamma' = -\frac{\xi^2}{a^2} sin\gamma_0 csc^2\gamma \bullet \gamma'. \tag{14}$$

The last equation has two solutions:

1. $\gamma' = 0$– constant parameter area
2. $\xi^2 = a^2\left(1 - \frac{sin^3\gamma}{sin\gamma_0}\right)$ – region of a self-similar (homogeneous) solution

Let us consider the first case

7. Stretching blow on the cylindrical net (Solution)

Let a semi-infinite unloaded cylindrical net be driven from the end with a constant velocity v. Since waves with greater deformation propagate with greater velocity, the wave front will undergo a jump.

Consider the motion of the net in the vicinity of the wave front [Fig. 1]:

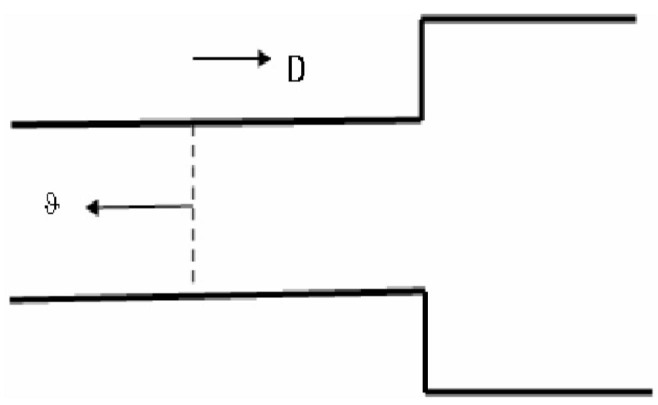

Fig. 1. Motion of the net in the vicinity of the wave front

In time dt, the front propagates to the distance Ddt. For a deformed net, there will be $(v + D)\,dt$. Denoting the values of density ρ_0 for an undeformed net, the law of conservation of mass will have the form [Fig. 2] and ρ for a deformed net

$$\rho\,(D + v) = \rho_0 D. \tag{15}$$

The change in momentum $\rho_0 Dvdt$ will be equal to the momentum of the force

$$\rho_0 Dv + 2\sigma cos\gamma = 0. \tag{16}$$

We connect the net densities with the deformation of the net element [Fig. 2].

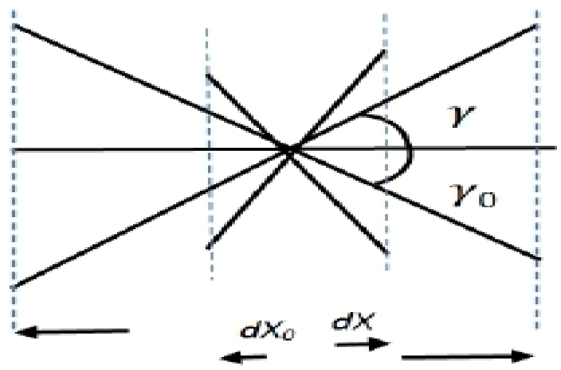

Fig. 2. Connection the deformation of the net with a density

If the mass of the net element dM, the deformation e, the slope angles of the branches in the initial and deformed state γ_0 and γ, then

$$\rho_0 = \frac{dM}{cos\gamma_0 dS} \quad ? \quad \rho = \frac{dM}{(1+e)\, cos\gamma dS}$$

or

$$\rho_0 = \frac{(1+e)\, cos\gamma}{cos\gamma_0}\rho \tag{17}$$

Substituting (17) into (15), we obtain

$$v = -\left[\frac{(1+e)\, cos\gamma}{cos\gamma_0} - 1\right] D \tag{18}$$

Substituting (18) into (16), we obtain

$$D^2 = \frac{\sigma cos\gamma_0}{\rho_0 \left[(1+e)\, cos\gamma - cos\gamma_0\right]} \tag{19}$$

Formulas (9), (18) and (19) allow to determine the shock wave velocity D, strain (tension) and turning angle of the net branches at a given impact speed.

It should be noted that with increasing impact velocity$v\gamma \to 0$, we have

$$D^2 = \frac{\sigma w_1 \gamma_0}{\rho_0 \left(1+e-cos\gamma_0\right)} \tag{20}$$

Setting $\sigma = Ee$, defining from (9)

$$1+e = \frac{sin\gamma_0}{sin\gamma}; \qquad \sigma = E\left(\frac{sin\gamma_0}{sin\gamma} - 1\right)$$

and substituting in (19) we obtain

$$D^2 = \frac{E\left(\frac{sin\gamma_0}{sin\gamma} - 1\right) cos\gamma_0}{\rho_0 \left(sin\gamma_0 ctg\gamma - cos\gamma_0\right)} \tag{21}$$

or

$$D^2 = a^2 \frac{\left(sin\gamma_0 - sin\gamma\right) cos\gamma_0}{sin\gamma_0 cos\gamma - cos\gamma_0 sin\gamma}$$

or

$$D = a\sqrt{\left(sin\gamma_0 - sin\gamma\right)/sin\left(\gamma_0 - \gamma\right)}$$

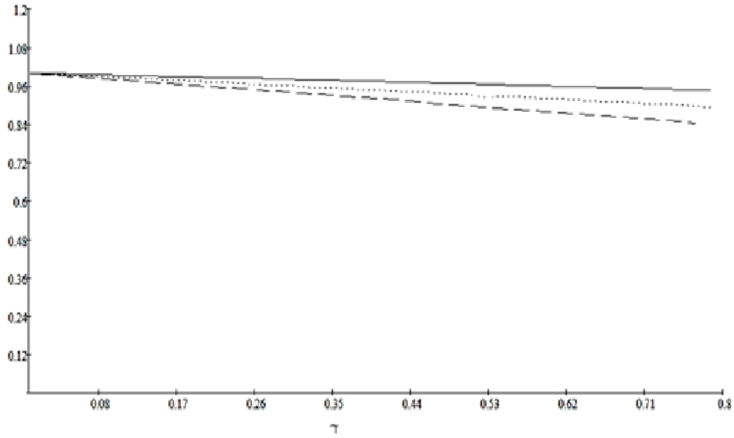

Fig. 3. $\gamma_0 = \frac{\pi}{4}; \gamma_0 = \frac{\pi}{6}; \gamma_0 = \frac{\pi}{12}$

Three variants of shock wave velocity distribution are calculated depending on the impact speed at the initial values of the angle of inclination of the branches of the net to the axis:$\frac{\pi}{4}$; $\frac{\pi}{6}$; $\frac{\pi}{12}$.

As can be seen from Graph 3, with increasing impact speed (decrease in γ), the shock wave velocity increases (up to 15%)

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