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Inverse Problem For A Third Order Hyperbolic Equation

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Abstract. In this work a nonlinear inverse boundary value problem for a hyperbolic equation of the third order is investigated. Using the Fourier method, the problem is reduced to solving a system of integral equations, and using the contraction mapping method, the existence and uniqueness of a solution to a system of integral equations are proved. The existence and uniqueness of the classical solution to the initial problem are proved.

Key Words and Phrases: hyperbolic equation, inverse problem, integral condition of overdetermination.

2010 Mathematics Subject Classifications: 35L25, 35R30

1. Introduction

There are many cases when the needs of practice lead to problems of determining the coefficients or the right-hand side of a differential equation according some known data of its solution. Such problems are called inverse problems of mathematical physics. Inverse problems are an actively developing branch of modern mathematics. Inverse problems for partial differential equations of various types were studied in many works [1–5]. In inverse problems, along with the initial and boundary conditions characteristic of a particular direct problem, additional information is given, the need for which is due to the presence of unknown coefficients or the right-hand side of the equation. Additional information, called an overdetermination condition, can be presented in various forms.

In the proposed article, an inverse boundary value problem with additional integral conditions for a third-order hyperbolic equation is studied.

2. Statement of the problem and its reduction to an equivalent problem

Let $D_T = \{(x,t): \ 0 \le x \le 1, \ 0 \le t \le T\}$. Next, let $f(x,t)$, $g(x,t)$, $\omega(x)$,$\varphi_i(x)$, $(i = 1, 2.3)$, $h_i(t)\,(i = 1,2)$ - be the given functions defined for $x \in [0,1]$, $t \in [0,T]$. Consider the following inverse boundary value problem: It is required to find the triple $\{u(x,t), a(t), b(t)\}$ of the functions $u(x,t), a(t), b(t)$ related by the equation [6]:

$$u_{ttt}(x,t) - u_{txx}(x,t) + u_{tt}(x,t) - \alpha u_{xx}(x,t) = a(t)u(x,t) + b(t)g(x,t) + f(x,t) \quad (1)$$

when the initial conditions are fulfilled for the function $u(x,t)$

$$u(x,0)=\varphi_0(x),\quad u_t(x,0)=\varphi_1(x),\quad u_{tt}(x,0)=\varphi_2(x)\quad (0\le x\le 1) \tag{2}$$

boundary conditions

$$u_x(0,t)=0,\quad u(1,t)=0\quad (0\le t\le T), \tag{3}$$

and with additional conditions

$$\int_0^1 \omega(x)u(x,t)dx \; = h_1(t)\;\; (0\le t\le T), \tag{4}$$

$$u(0,t)=h_2(t)\quad (0\le t\le T), \tag{5}$$

where $0<\alpha<1$ – is a given number.

Denote

$$\tilde{C}^{(2,3)}\left(D_T\right)=\{u(x,t)\;,\;u_x(x,t),u_{xx}(x,t),u_t(x,t),u_{tx}(x,t),u_{txx}(x,t),$$

$$u_{tt}(x,t),u_{ttt}(x,t)\in C\left(D_T\right)\}\,.$$

Definition 1. *By the classical solution of the inverse boundary value problem* (1) - (5) *we mean a triple* $\{u(x,t),a(t),b(t)\}$ *of functions* $u\left(x,t\right)\in\tilde{C}^{(2,3)}\left(D_T\right)$, $a\left(t\right)\in C[0,T]$, $b\left(t\right)\in C[0,T]$, *satisfying equation* (1) *and conditions* (2) - (5) *in the usual sense.*

Similarly to [7], the following theorem is proved.

Theorem 1. *Let* $f(x,t),g(x,t)\in C(D_T)$, $\varphi_i(x)\in C[0,1]$ $(i=1,2,3)$,$h_i\left(t\right)\in C^3\left[0,T\right]$ $(i=1,2)$ *and the conditions of agreement are fulfilled:*

$$\int_0^1\omega(x)\varphi_0\left(x\right)dx=h_1\left(0\right),\;\int_0^1\omega(x)\varphi_1\left(x\right)dx=h_1'\left(0\right),\;\int_0^1\omega(x)\varphi_2\left(x\right)dx=h_1''\left(0\right)\;,$$

$$\varphi_0\left(0\right)=h_2\left(0\right),\;\;\varphi_1\left(0\right)=h_2'\left(0\right)\;,\;\;\varphi_2\left(0\right)=h_2''\left(0\right)\;.$$

Then the problem of finding a classical solution to problem (1) - (5) is equivalent to the problem of determining functions $u\left(x,t\right)\in\tilde{C}^{(2,3)}\left(D_T\right)$, $a\left(t\right)\in C[0,T]$, $b\left(t\right)\in C[0,T]$ from relations (1) - (3) and

$$a(t)h_1(t)\;+\;b(t)\int_0^1\omega(x)g\left(x,t\right)dx=$$

$$=h_1'''(t)-\int_0^1\omega(x)f\left(x,t\right)dx-\int_0^1\omega(x)u_{txx}(x,t)dx\;+h_1''(t)-\alpha\int_0^1\omega(x)u_{xx}(x,t)dx\;\;, \tag{6}$$

$$a(t)h_2(t)\left(t\right)\;+\;b(t)g(0,t)=h_2'''(t)\;-f(0,t)-\;u_{txx}(0,t)\;+\;h_2''(t)\;-\;\alpha u_{xx}(0,t)\,, \tag{7}$$

moreover

$$h\left(t\right)\equiv h_1(t)\,g\left(0,t\right)-h_2(t)\int_0^1\omega(x)g\left(x,t\right)dx\quad (0\le t\le T).$$

3. Solvability of the problem

The first component $u(x,t)$ of the solution $\{u(x,t), a(t), b(t)\}$ to problem (1) - (3), (6), (7) will be sought in the form

$$u(x,t)=\sum_{k=1}^{\infty} u_k(t)\cos\lambda_k x,\ \ \lambda_k=\frac{\pi}{2}(2k-1), \tag{8}$$

where

$$u_k(t)=2\int_0^1 u(x,t)\cos\lambda_k x dx\ (k=1,2,...).$$

Then, applying the formal Fourier scheme, from (1) and (2) we have:

$$u_k'''(t)+u_k''(t)+\lambda_k^2 u_k'(t)+\alpha\lambda_k^2 u_k(t)=F_k(t;u,a,b)\ \ (k=1,2,...;\ \ 0\le t\le T), \tag{9}$$

$$u_k(0)=\varphi_{0k},\ \ u_k'(0)=\varphi_{1k},\ \ u_k''(0)=\varphi_{2k}\ \ (k=1,2,...), \tag{10}$$

where

$$F_k(t;u,a,b)=f_k(t)+a(t)u_k(t)+b(t)g_k(t), f_k(t)=2\int_0^1 f(x,t)\cos\lambda_k x dx,$$

$$g_k(t)=2\int_0^1 g(x,t)\cos\lambda_k x dx,\ \ \varphi_{ik}=2\int_0^1 \varphi_i(x)\cos\lambda_k x\, dx(i=0,1,2;\ \ k=1,2,...).$$

Solving problem (9), (10), we find:

$$\begin{aligned} u_k(t)=\frac{1}{b_k}\Bigg\{\Bigg[(\gamma_k^2+\beta_k^2)e^{\alpha_k t}+e^{\gamma_k t}\Big[\alpha_k(\alpha_k-2\gamma_k)\cos\beta_k t+ \\ +\frac{1}{\beta_k}(\gamma_k^3+\alpha_k\gamma_k^2-\alpha_k\beta_k^2-\alpha_k^2\gamma_k)\sin\beta_k t\Big]\Bigg]\varphi_{0k}+ \\ +\Bigg[-2\gamma_k e^{\alpha_k t}+e^{\gamma_k t}\Big[2\gamma_k\cos\beta_k t+\frac{1}{\beta_k}(\alpha_k^2+\beta_k^2-\gamma_k^2)\sin\beta_k t\Big]\Bigg]\varphi_{1k}+ \\ +\Bigg[e^{\alpha_k t}+e^{\gamma_k t}\Big[-\cos\beta_k t+\frac{1}{\beta_k}(\gamma_k-\alpha_k)\sin\beta_k t\Big]\Bigg]\varphi_{2k}+\int_0^t F_k(\tau;u,a,b)\times \\ \times\Bigg[e^{\alpha_k(t-\tau)}+e^{\gamma_k(t-\tau)}\Big[\frac{\gamma_k-\alpha_k}{\beta_k}\sin\beta_k(t-\tau)-\cos\beta_k(t-\tau)\Big]\Bigg]d\tau\Bigg\}\ \ (k=1,2,...), \end{aligned} \tag{11}$$

where

$$\alpha_k=\alpha_{1k}+\beta_{1k}-\frac{1}{3},\ \ \beta_k=\frac{\sqrt{3}}{2}(\alpha_{1k}-\beta_{1k}),\ \ \gamma_k=-\frac{1}{3}-\frac{1}{2}(\alpha_{1k}+\beta_{1k}),$$

$$b_k=\alpha_k^2+\beta_k^2+\gamma_k^2-2\alpha_k\gamma_k,$$

moreover

$$\alpha_{1k}=\left\{-\frac{1}{2}\left(\left(\alpha-\frac{1}{3}\right)\lambda_k^2+\frac{2}{27}\right)+\left[\frac{1}{4}\left(\left(\alpha-\frac{1}{3}\right)\lambda_k^2+\frac{2}{27}\right)^2+\frac{1}{27}\left(\lambda_k^2-\frac{1}{3}\right)^3\right]^{1/2}\right\}^{1/3}, \tag{12}$$

$$\beta_{1k}=\left\{-\frac{1}{2}\left(\left(\alpha-\frac{1}{3}\right)\lambda_k^2+\frac{2}{27}\right)-\left[\frac{1}{4}\left(\left(\alpha-\frac{1}{3}\right)\lambda_k^2+\frac{2}{27}\right)^2+\frac{1}{27}\left(\lambda_k^2-\frac{1}{3}\right)^3\right]^{1/2}\right\}^{1/3}. \tag{13}$$

After substituting the expressions$u_k(t)$ $(k=1,2,...)$ in (8), to determine the components $u(x,t)$ of the solution to problem (1) - (3), (6), (7), we obtain:

$$u(x,t)=\sum_{k=1}^{\infty}\left\{\frac{1}{b_k}\left\{\left[(\gamma_k^2+\beta_k^2)e^{\alpha_k t}+e^{\gamma_k t}\left[\alpha_k(\alpha_k-2\gamma_k)\cos\beta_k t+\right.\right.\right.\right.$$
$$\left.\left.+\frac{1}{\beta_k}(\gamma_k^3+\alpha_k\gamma_k^2-\alpha_k\beta_k^2-\alpha_k^2\gamma_k)\sin\beta_k t\right]\right]\varphi_{0k}+$$
$$+\left[-2\gamma_k e^{\alpha_k t}+e^{\gamma_k t}\left[2\gamma_k\cos\beta_k t+\frac{1}{\beta_k}(\alpha_k^2+\beta_k^2-\gamma_k^2)\sin\beta_k t\right]\right]\varphi_{1k}+$$
$$+\left[e^{\alpha_k t}+e^{\gamma_k t}\left[-\cos\beta_k t+\frac{1}{\beta_k}(\gamma_k-\alpha_k)\sin\beta_k t\right]\right]\varphi_{2k}+\int_0^t F_k(\tau;u,a,b)\times$$
$$\left.\left.\times\left[e^{\alpha_k(t-\tau)}+e^{\gamma_k(t-\tau)}\left[\frac{\gamma_k-\alpha_k}{\beta_k}\sin\beta_k(t-\tau)-\cos\beta_k(t-\tau)\right]\right]d\tau\right\}\right\}\cos\lambda_k x. \tag{14}$$

Differentiating (13) we find:

$$u_k'(t)=\frac{1}{b_k}\left\{\left[\alpha_k(\gamma_k^2+\beta_k^2)e^{\alpha_k t}+e^{\gamma_k t}\left[-\alpha_k(\gamma_k^2+\beta_k^2)\cos\beta_k t+\frac{\alpha_k}{\beta_k}(\gamma_k-\alpha_{kk})\times\right.\right.\right.$$
$$\left.\left.\times(\gamma_k^2+\beta_k^2)\sin\beta_k t\right]\right]\varphi_{0k}+\left[-2\alpha_k\gamma_k e^{\alpha_k t}+e^{\gamma_k t}\left[(\alpha_k^2+\beta_k^2+\gamma_k^2)\cos\beta_k t+\right.\right.$$
$$\left.\left.+\frac{\gamma_k}{\beta_k}(\alpha_k^2-\beta_k^2-\gamma_k^2)\sin\beta_k t\right]\right]\varphi_{1k}+\left[\alpha_k e^{\alpha_k t}+e^{\gamma_k t}\left[-\alpha_k\cos\beta_k t+\right.\right.$$
$$\left.\left.+\frac{1}{\beta_k}(\beta_k^2+\gamma_k^2-\alpha_k\gamma_k)\sin\beta_k t\right]\right]\varphi_{2k}+\int_0^t F_k(\tau;u,a,b)\left[\alpha_k e^{\alpha_k(t-\tau)}+e^{\gamma_k(t-\tau)}\times\right.$$
$$\left.\left.\times\left[\left(\frac{\gamma_k}{\beta_k}(\gamma_k-\alpha_k)+\beta_k\right)\sin\beta_k(t-\tau)-\alpha_k\cos\beta_k(t-\tau)\right]\right]d\tau\right\}\quad(k=1,2,...). \tag{15}$$

Now from (6) and (7) taking into account (8), respectively, we have:

$$a(t)h_1(t)+b(t)\int_0^1\omega(x)g(x,t)\,dx=$$

$$= h_1'''(t) \ + h_1''(t) - \int_0^1 \omega(x) f\,(x,t)\,dx + \sum_{k=1}^{\infty} \lambda_k^2 (u_k'(t) \ + \ \alpha u_k(t)) \int_0^1 \omega(x) \cos \lambda_x x dx \ , \quad (16)$$

$$a(t) h_2(t)\,(t) \ + \ b(t) g(0,t) = h_2'''(t) \ + \ h_2''(t) \ - f(0,t) + \sum_{k=1}^{\infty} \lambda_k^2 (u_k'(t) \ + \ \alpha u_k(t))\,. \quad (17)$$

Suppose that

$$h\,(t) \equiv h_1(t)\, g\,(0,t) - h_2(t) \int_0^1 \omega(x) g\,(x,t)\,dx \ \neq 0 \ \ (0 \leq t \leq T).$$

Then from (16) and (17) we obtain:

$$a(t) = [h(t)]^{-1} \left\{ \left(h_1'''(t) + \ h_1''\,(t) \ - \int_0^1 \omega(x) f\,(x,t)\,dx \right) g(0,t) - \right.$$

$$- \left(h_2'''(t) + \ h_2''\,(t) \ - f(0,t) \right) \int_0^1 \omega(x) g\,(x,t)\,dx +$$

$$\left. + \sum_{k=1}^{\infty} \lambda_k^2 (u_k'(t) \ + \ \alpha u_k(t)) \left(g(0,t) \int_0^1 \omega(x) \cos \lambda_k dx - \int_0^1 \omega(x) g\,(x,t)\,dx \right) \right\} \ , \quad (18)$$

$$b(t) = [h(t)]^{-1} \left\{ \left(h_2'''(t) + \ h_2''\,(t) \ - f\,(0,t) \right) \ h_1\,(t) \ - \right.$$

$$\left(h_1'''(t) + \ h_1''\,(t) \ - f(0,t) - \int_0^1 \omega(x) f\,(x,t)\,dx \right) h_2\,(t) \ +$$

$$\left. + \sum_{k=1}^{\infty} \lambda_k^2 (u_k'(t) \ + \ \alpha u_k(t)) \left(h_1\,(t) - h_2\,(t) \int_0^1 \omega(x) \cos \lambda_k dx \right) \right\} \ . \quad (19)$$

Further, from (11) and (15), we obtain:

$$u_k'(t) \ + \ \alpha u_k(t) = \frac{1}{b_k} \left\{ \left[(\alpha \ + \ \alpha_k)(\gamma_k^2 \ + \ \beta_k^2) e^{\alpha_k t} \ + \ e^{\gamma_k t} \left[\alpha_k (\alpha \alpha_k \ - \ 2\alpha\gamma_k \ - \ \gamma_k^2 \ - \ \beta_k^2) \times \right. \right. \right.$$

$$\left. \left. \times \cos \beta_k t \ + \frac{\alpha_k}{\beta_k} ((\gamma_k - \alpha_k)(\gamma_k^2 + \beta_k^2) + \alpha(\gamma_k^2 - \beta_k^2 - \alpha_k \gamma_k)) \sin \beta_k t \right] \right] \varphi_{0k} +$$

$$+ \left[-2(\alpha + \alpha_k)\gamma_k e^{\alpha_k t} + e^{\gamma_k t} \left[(2\alpha\gamma_k + \alpha_k^2 + \beta_k^2 + \gamma_k^2) \cos \beta_k t + \right. \right.$$

$$\left. \left. + \frac{1}{\beta_k} (\alpha(\alpha_k^2 + \beta_k^2 - \gamma_k^2) + \gamma_k (\alpha_k^2 - \beta_k^2 - \gamma_k^2)) \sin \beta_k t \right] \right] \varphi_{1k} + \left[(\alpha + \alpha_k) e^{\alpha_k t} + \right.$$

$$\left. + \ e^{\gamma_k t} \left[-(\alpha + \alpha_k) \cos \beta_k t + \frac{1}{\beta_k} (\alpha\gamma_k - \alpha\alpha_k + \beta_k^2 + \gamma_k^2 - \alpha_k \gamma_k) \sin \beta_k t \right] \right] \varphi_{2k} +$$

$$+\int_0^t F_k(\tau;u,a,b)\left[(\alpha+\alpha_k)e^{\alpha_k(t-\tau)}+e^{\gamma_k(t-\tau)}\left[(-\alpha+\alpha_k)\cos\beta_k(t-\tau)+\right.\right.$$

$$\left.\left.+\left(\frac{\gamma_k+\alpha}{\beta_k}(\gamma_k-\alpha_k)+\beta_k\right)\sin\beta_k(t-\tau)\right]\right]d\tau\Bigg\}\quad (k=1,2,...)\ . \tag{20}$$

In order to obtain an equation for the second and third components of the solution $\{u(x,t),a(t),b(t)\}$ of problem (1) - (3), (6), (7), we substitute expression (20) into (18) and (19) :

$$a(t)=[h(t)]^{-1}\left\{\left(h_1'''(t)+h_1''(t)-\int_0^1\omega(x)f(x,t)\,dx\right)g(0,t)-\right.$$

$$-\left(h_2'''(t)+h_2''(t)-f(0,t)\right)\int_0^1\omega(x)g(x,t)\,dx+$$

$$+\sum_{k=1}^{\infty}\frac{\lambda_k^2}{b_k}\left\{\left[\left[(\alpha+\alpha_k)(\gamma_k^2+\beta_k^2)e^{\alpha_k t}+e^{\gamma_k t}\left[\alpha_k(\alpha\alpha_k-2\alpha\gamma_k-\gamma_k^2-\beta_k^2)\times\right.\right.\right.\right.$$

$$\left.\left.\times\cos\beta_k t+\frac{\alpha_k}{\beta_k}((\gamma_k-\alpha_k)(\gamma_k^2+\beta_k^2)+\alpha(\gamma_k^2-\beta_k^2-\alpha_k\gamma_k))\sin\beta_k t\right]\right]\varphi_{0k}+$$

$$+\left[-2(\alpha+\alpha_k)\gamma_k e^{\alpha_k t}+e^{\gamma_k t}\left[(2\alpha\gamma_k+\alpha_k^2+\beta_k^2+\gamma_k^2)\cos\beta_k t+\right.\right.$$

$$\left.\left.+\frac{1}{\beta_k}(\alpha(\alpha_k^2+\beta_k^2-\gamma_k^2)+\gamma_k(\alpha_k^2-\beta_k^2-\gamma_k^2))\sin\beta_k t\right]\right]\varphi_{1k}+\left[(\alpha+\alpha_k)e^{\alpha_k t}+\right.$$

$$\left.+e^{\gamma_k t}\left[-(\alpha+\alpha_k)\cos\beta_k t+\frac{1}{\beta_k}(\alpha\gamma_k-\alpha\alpha_k+\beta_k^2+\gamma_k^2-\alpha_k\gamma_k)\sin\beta_k t\right]\right]\varphi_{2k}+$$

$$+\int_0^t F_k(\tau;u,a,b)\left[(\alpha+\alpha_k)e^{\alpha_k(t-\tau)}+e^{\gamma_k(t-\tau)}\left[(-\alpha+\alpha_k)\cos\beta_k(t-\tau)+\right.\right.$$

$$\left.\left.+\left(\frac{\gamma_k+\alpha}{\beta_k}(\gamma_k-\alpha_k)+\beta_k\right)\sin\beta_k(t-\tau)\right]\right]d\tau\Bigg\}$$

$$\left.\left(g(0,t)\int_0^1\omega(x)\cos\lambda_k x dx-\int_0^1\omega(x)g(x,t)\,dx\right)\right\}, \tag{21}$$

$$b(t)=[h(t)]^{-1}\left\{\left(h_2'''(t)+h_2''(t)-f(0,t)\right)h_1(t)-\right.$$

$$\left(h_1'''(t)+h_1''(t)-f(0,t)-\int_0^1\omega(x)f(x,t)\,dx\right)h_2(t)+$$

$$+\sum_{k=1}^{\infty}\frac{\lambda_k^2}{b_k}\left\{\left[\left[(\alpha+\alpha_k)(\gamma_k^2+\beta_k^2)e^{\alpha_k t}+e^{\gamma_k t}\left[\alpha_k(\alpha\alpha_k-2\alpha\gamma_k-\gamma_k^2-\beta_k^2)\times\right.\right.\right.\right.$$

$$\times \cos \beta_k t + \frac{\alpha_k}{\beta_k}((\gamma_k - \alpha_k)(\gamma_k^2 + \beta_k^2) + \alpha(\gamma_k^2 - \beta_k^2 - \alpha_k\gamma_k)) \sin \beta_k t\Big]\Big] \varphi_{0k} +$$

$$+ \Big[-2(\alpha + \alpha_k)\gamma_k e^{\alpha_k t} + e^{\gamma_k t}\Big[(2\alpha\gamma_k + \alpha_k^2 + \beta_k^2 + \gamma_k^2) \cos \beta_k t +$$

$$+ \frac{1}{\beta_k}(\alpha(\alpha_k^2 + \beta_k^2 - \gamma_k^2) + \gamma_k(\alpha_k^2 - \beta_k^2 - \gamma_k^2)) \sin \beta_k t\Big]\Big] \varphi_{1k} + \Big[(\alpha + \alpha_k)e^{\alpha_k t} +$$

$$+ e^{\gamma_k t}\Big[-(\alpha + \alpha_k) \cos \beta_k t + \frac{1}{\beta_k}(\alpha\gamma_k - \alpha\alpha_k + \beta_k^2 + \gamma_k^2 - \alpha_k\gamma_k) \sin \beta_k t\Big]\Big] \varphi_{2k} +$$

$$+ \int_0^t F_k(\tau; u, a, b)\Big[(\alpha + \alpha_k)e^{\alpha_k(t-\tau)} + e^{\gamma_k(t-\tau)}\Big[(-\alpha + \alpha_k) \cos \beta_k(t - \tau) +$$

$$+ \left(\frac{\gamma_k + \alpha}{\beta_k}(\gamma_k - \alpha_k) + \beta_k\right) \sin \beta_k(t - \tau)\Big]\Big] d\tau\Big\} \left(h_1(t) - h_2(t) \int_0^1 \omega(x) \cos \lambda_k x dx\right) \Big\}. \tag{22}$$

Thus, the solution of problem (1) - (3), (6), (7) is reduced to the solution of system (14), (21), (22) with respect to unknown functions $u(x,t)$, $a(t)$ and $b(t)$.

The following lemma is true.

Lemma 1. *If* $\{u(x,t), a(t), b(t)\}$ *–is any classical solution to problem* (1) (1)-(3), (6), (7), *then the functions*

$$u_k(t) = 2 \int_0^1 u(x,t) \cos \lambda_k x dx \quad (k = 1, 2, ...)$$

satisfy the system (11).

Corollary 1. *Lemma 1 implies that to prove the uniqueness of the solution to problem* (1) - (3), (6), (7), *it suffices to prove the uniqueness of the solution to system* (14), (21), (22).

Now, in order to study problem (1) - (3), (6), (7), consider the following spaces:

1. Let us denote by $B_{2,T}^3$ [8] the collection of all functions $u(x,t)$ of the form

$$u(x_i t) = \sum_{k=1}^{\infty} u_k(t) \cos \lambda_k x, \lambda_k = \frac{\pi}{2}(2k - 1),$$

considered inD_T for which all functions $u_k(t) \in C[0,T]$ and

$$J_T(u) \equiv \left(\sum_{k=1}^{\infty} (\lambda_k^3 \| u_k(t) \|_{C[0,T]})^2\right)^{1/2} < \infty.$$

The norm in this set is defined as follows:

$$\|u(x,t)\|_{B_{2,T}^{3,}} = J_T(u).$$

2. Let us denote by E_T^3 the spaces of the vector of functions $\{u(x,t), a(t), b(t)\}$ such that

$$u(x,t) \in B_{2,T}^3, a(t) \in C[0,T], b(t) \in C[0,T] .$$

We equip this space with a norm:

$$\|z\|_{E_T^3} = \|u(x,t)\|_{B_{2,T}^3} + \|a(t)\|_{C[0,T]} + \|b(t)\|_{C[0,T]} .$$

It is known that $B_{2,T}^3$ and E_T^3 are Banach spaces.

Consider the following operator in space E_T^3

$$\Phi(u,a,b) = \{\Phi_1(u,a,b), \Phi_2(u,a,b), \Phi_3(u,a,b)\},$$

where

$$\Phi_1(u,a,b) = \tilde{u}(x,t) \equiv \sum_{k=1}^{\infty} \tilde{u}_k(t) \cos \lambda_k x, , \Phi_2(u,a,b) = \tilde{a}(t), \ , \Phi_3(u,a,b) = \tilde{b}(t)$$

where $\tilde{u}_k(t)$ $(k = 1,2,...)$, $\tilde{a}(t)$ and $\tilde{b}(t)$ are equal, respectively, to the right-hand sides (11), (21) and (22).

Accept the notation

$$\alpha_{2k} = -\frac{1}{2}\left(\left(\alpha - \frac{1}{3}\right)\lambda_k^2 + \frac{2}{27}\right) + \left[\frac{1}{4}\left(\left(\alpha - \frac{1}{3}\right)\lambda_k^2 + \frac{2}{27}\right)^2 + \frac{1}{27}\left(\lambda_k^2 - \frac{1}{3}\right)^3\right]^{1/2}, \quad (23)$$

$$\beta_{2k} = \frac{1}{2}\left(\left(\alpha - \frac{1}{3}\right)\lambda_k^2 + \frac{2}{27}\right) + \left[\frac{1}{4}\left(\left(\alpha - \frac{1}{3}\right)\lambda_k^2 + \frac{2}{27}\right)^2 + \frac{1}{27}\left(\lambda_k^2 - \frac{1}{3}\right)^3\right]^{1/2}. \quad (24)$$

Then

$$\alpha_{1k} = \sqrt[3]{\alpha_{2k}}, \quad \beta_{1k} = -\sqrt[3]{\beta_{2k}} .$$

Hence, taking into account (23) and (24), we obtain:

$$\alpha_{1k} + \beta_{1k} = \left|\sqrt[3]{\alpha_{2k}} - \sqrt[3]{\beta_{2k}}\right| = \left|\frac{\alpha_{2k} - \beta_{2k}}{\sqrt[3]{\alpha_{2k}^2} + \sqrt[3]{\alpha_{2k}\beta_{2k}} + \sqrt[3]{\beta_{2k}^2}}\right| \le \frac{9\alpha}{2} + \frac{11}{2}.$$

It is easy to see that

$$|\alpha_k| \le \left|\alpha_{1k} + \beta_{1k} - \frac{1}{3}\right| \le \frac{9\alpha}{2} + \frac{13}{6} \equiv \varepsilon_1 , |\gamma_k| = \left|-\frac{1}{3} - \frac{\alpha_{1k} + \beta_{1k}}{2}\right| \le \frac{9\alpha}{4} + \frac{5}{4} \equiv \varepsilon_2 ,$$

$$\varepsilon_3\lambda_k \equiv \frac{\sqrt{2}}{3}\lambda_k \le \beta_k \le \sqrt[3]{\frac{1}{2}\left(\alpha - \frac{1}{27}\right) + \sqrt{\frac{1}{4}\left(\alpha - \frac{1}{27}\right)^2 + \frac{1}{27}}}\ {}_k\lambda_k \equiv \varepsilon_4\lambda_k ,$$

$$b_k = (\alpha_k - \gamma_k)^2 + \beta_k^2 \geq \beta_k^2 \geq \varepsilon_3^2 \lambda_k^2 \,,$$

Taking these relations into account, we find:

$$\left(\sum_{k=1}^{\infty} (\lambda_k^3 \left\| \tilde{u}_k(t) \right\|_{C[0,T]})^2 \right)^{1/2} \leq \rho_0(T) \left(\sum_{k=1}^{\infty} (\lambda_k^3 \,|\, \varphi_{0k} \,|)^2 \right)^{1/2} + \rho_1(T) \left(\sum_{k=1}^{\infty} (\lambda_k^3 \,|\, \varphi_{1k} \,|)^2 \right)^{1/2} +$$

$$+\rho_2(T) \left(\sum_{k=1}^{\infty} (\lambda_k^2 \,|\, \varphi_{2k} \,|)^2 \right)^{1/2} + \rho_2(T)\sqrt{T} \left(\int_0^T \sum_{k=1}^{\infty} (\lambda_k^2 \,|\, f_k(\tau) \,|)^2 d\tau \right)^{1/2} +$$

$$+\rho_2(T) T \left\| a(t) \right\|_{C[0,T]} \left(\sum_{k=1}^{\infty} (\lambda_k^3 \left\| u_k(t) \right\|_{C[0,T]})^2 \right)^{1/2} ,$$

$$+\rho_2(T)\sqrt{T} \left\| b(t) \right\|_{C[0,T]} \left(\int_0^T \sum_{k=1}^{\infty} (\lambda_k^2 \,|\, g_k(\tau) \,|)^2 d\tau \right)^{1/2} , \tag{25}$$

$$\left\| \tilde{a}(t) \right\|_{C[0,T]} \leq \left\| [h(t)]^{-1} \right\|_{C[0,T]} \times$$

$$\times \left\{ \left\| \left(h_1'''(t) + h_1''(t) - \int_0^1 \omega(x) f(x,t)\, dx \right) g(0,t) - \right.\right.$$

$$- \left(h_2'''(t) + h_2''(t) - f(0,t) \right) \int_0^1 \omega(x) g(x,t)\, dx \Bigg\|_{C[0,T]} +$$

$$+ \left(\sum_{k=1}^{\infty} \lambda_k^{-2} \right)^{1/2} \left(\left\| g(0,t) \right\|_{C[0,T]} \left\| \omega(x) \right\|_{L_2(0,1)} + \left\| \int_0^1 \omega(x) g(x,t)\, dx) \right\|_{C[0,T]} \right) \times$$

$$\times \left[\rho_3(T) \left(\sum_{k=1}^{\infty} (\lambda_k^3 \,|\, \varphi_{0k} \,|)^2 \right)^{1/2} + \rho_4(T) \left(\sum_{k=1}^{\infty} (\lambda_k^3 \,|\, \varphi_{1k} \,|)^2 \right)^{1/2} - \right.$$

$$+\rho_5(T) \left(\sum_{k=1}^{\infty} (\lambda_k^2 \,|\varphi_{2k}|)^2 \right)^{1/2} + \rho_5(T)\sqrt{T} \left(\int_0^T \sum_{k=1}^{\infty} (\lambda_k^2 \,|f_k(\tau)|)^2 d\tau \right)^{1/2} +$$

$$+\rho_5(T) T \left\| a(t) \right\|_{C[0,T]} \left(\sum_{k=1}^{\infty} (\lambda_k^3 \left\| u_k(t) \right\|_{C[0,T]})^2 \right)^{1/2} +$$

$$\left.\left. +\rho_5(T)\sqrt{T} \left\| b(t) \right\|_{C[0,T]} \left(\int_0^T \sum_{k=1}^{\infty} (\lambda_k^2 \,|g_k(\tau)|)^2 d\tau \right)^{1/2} \right] \right\} , \tag{26}$$

$$\left\| \tilde{b}(t) \right\|_{C[0,T]} \leq \left\| [h(t)]^{-1} \right\|_{C[0,T]} \times$$

$$\times\left\{\left\|\left(h_2'''(t)+h_2''(t)-f(0,t)\right)h_1(t)-\right.\right.$$

$$\left.-\left(h_1'''(t)+h_1''(t)-f(0,t)-\int_0^1\omega(x)f(x,t)\,dx\right)h_2(t)\right\|_{C[0,T]}+$$

$$+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|h_1(t)\|_{C[0,T]}+\|h_2(t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}\right)\times$$

$$\times\left[\rho_3(T)\left(\sum_{k=1}^{\infty}(\lambda_k^3\,|\,\varphi_{0k}\,|)^2\right)^{1/2}+\rho_4(T)\left(\sum_{k=1}^{\infty}(\lambda_k^3\,|\,\varphi_{1k}\,|)^2\right)^{1/2}-\right.$$

$$+\rho_5(T)\left(\sum_{k=1}^{\infty}(\lambda_k^2\,|\varphi_{2k}|)^2\right)^{1/2}+\rho_5(T)\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}(\lambda_k^2\,|f_k(\tau)|)^2d\tau\right)^{1/2}+$$

$$+\rho_5(T)T\,\|a(t)\|_{C[0,T]}\left(\sum_{k=1}^{\infty}(\lambda_k^3\,\|u_k(t)\|_{C[0,T]})^2\right)^{1/2}+$$

$$\left.\left.+\rho_5(T)\sqrt{T}\,\|b(t)\|_{C[0,T]}\left(\int_0^T\sum_{k=1}^{\infty}(\lambda_k^2\,|g_k(\tau)|)^2d\tau\right)^{1/2}\right]\right\}, \tag{27}$$

where

$$\rho_0(T)=\frac{\sqrt{6}}{\varepsilon_3^2}\left\{(\varepsilon_2^2+\varepsilon_4^2)e^{\varepsilon_1 T}+\varepsilon_1 e^{\varepsilon_2 T}\left[\varepsilon_1+2\varepsilon_2+\frac{1}{\varepsilon_3}(\varepsilon_2^2+\varepsilon_4^2+\varepsilon_1\varepsilon_3)\right]\right\},$$

$$\rho_1(T)=\frac{\sqrt{6}}{\varepsilon_3^2}\left\{2\varepsilon_2 e^{\varepsilon_1 T}+_1 e^{\varepsilon_2 T}\left[2\varepsilon_2+\frac{1}{\varepsilon_3}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_4^2)\right]\right\},$$

$$\rho_2(T)=\frac{\sqrt{6}}{\varepsilon_3^2}\left\{e^{\varepsilon_1 T}+e^{\varepsilon_2 T}\left[1+\frac{1}{\varepsilon_3}(\varepsilon_1+\varepsilon_2)\right]\right\},$$

$$\rho_3(T)=\frac{1}{\varepsilon_3^2}\left\{(\alpha+\varepsilon_1)(\varepsilon_2^2+\varepsilon_4^2)e^{\varepsilon_1 T}+e^{\varepsilon_2 T}\left[\varepsilon_1(\alpha\varepsilon_1+2\alpha\varepsilon_2+\right.\right.$$

$$\left.\left.+\varepsilon_2^2+\varepsilon_4^2+\frac{\varepsilon_2}{\varepsilon_3}((\varepsilon_1+\varepsilon_2)(\varepsilon_2^2+\varepsilon_4^2)+\alpha(\varepsilon_2^2+\varepsilon_4^2+\varepsilon_1\varepsilon_3))\right]\right\},$$

$$\rho_4(T)=\frac{1}{\varepsilon_3^2}\left\{2\varepsilon_2(\alpha+\varepsilon_1)e^{\varepsilon_1 T}+e^{\varepsilon_2 T}\left[\varepsilon_1^2+\varepsilon_2^2+\varepsilon_4^2+\right.\right.$$

$$\left.\left.+2\alpha\varepsilon_2+\frac{1}{\varepsilon_3}(\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_4^2)+\alpha(\varepsilon_1^2+\varepsilon_4^2+\varepsilon_1\varepsilon_3))\right]\right\},$$

$$\rho_5(T)=\frac{1}{\varepsilon_3^2}\left\{(\alpha+\varepsilon_1)e^{\varepsilon_1 T}+e^{\varepsilon_2 T}\left[\alpha+\varepsilon_1+\frac{1}{\varepsilon_3}(\alpha\varepsilon_2+\alpha\varepsilon_1+\varepsilon_2^2+\varepsilon_4^2+\varepsilon_1\varepsilon_2)\right]\right\}.$$

Suppose that the given problem (1) - (3), (6), (7) satisfy the following conditions:

1. $\varphi_i(x) \in C^2[0,1],\ \varphi_i'''(x) \in L_2(0,1)$ and $\varphi_i'(0) = \varphi_i(1) = \varphi_i''(1) = 0\ \ (i = 0,1)$.
2. $\varphi_2(x) \in C^1[0,1],\ \varphi_2''(x) \in L_2(0,1)$ and $\varphi_2'(0) = \varphi_2(1) = 0$.
3. $f(x,t),\ f_x(x,t) \in C(D_T),\ f_{xx}(x,t) \in L_2(D_T)$ and $f_x(0,t) = f(1,t) = 0\ \ (0 \le t \le T)$.
4. $g(x,t),\ g_x(x,t) \in C(D_T),\ g_{xx}(x,t) \in L_2(D_T)$ and $g_x(0,t) = g(1,t) = 0\ \ (0 \le t \le T)$.
5. $h(t) \in C^3[0,T]$ $, h(t) \equiv h_1(t)\, g(0,t) - h_2(t) \int_0^1 \omega(x) g(x,t)\, dx \ne 0\ (0 \le t \le T), \omega(x) \in L_2(0,1)$.

Then from (25) - (27) we have:

$$\|\tilde{u}(x,t)\|_{B^3_{2,T}} \le A_1(T) + B_1(T)\,\|a(t)\|_{C[0,T]}\,\|u(x,t)\|_{B^{3,}_{2,T}} + C_1(T)\,\|b(t)\|_{C[0,T]}\,, \tag{28}$$

$$\|\tilde{a}(t)\|_{C[0,T]} \le A_2(T) + B_2(T)\,\|a(t)\|_{C[0,T]}\,\|u(x,t)\|_{B^3_{2,T}} + C_3(T)\,\|b(t)\|_{C[0,T]}\,, \tag{29}$$

$$\left\|\tilde{b}(t)\right\|_{C[0,T]} \le A_3(T) + B_3(T)\,\|a(t)\|_{C[0,T]}\,\|u(x,t)\|_{B^3_{2,T}} + C_3(T)\,\|b(t)\|_{C[0,T]}\,. \tag{30}$$

where

$$A_1(T) = \rho_0(T)\left\|\varphi_0'''(x)\right\|_{L_2(0,1)} + \rho_1(T)\left\|\varphi_1'''(x)\right\|_{L_2(0,1)} +$$
$$+\rho_2(T)\left\|\varphi_2''(x)\right\|_{L_2(0,1)} + \rho_2(T)\sqrt{T}\,\|f_{xx}(x,t)\|_{L_2(D_T)}\,,$$
$$B_1(T) = \rho_2(T)T, C_1(T) = \rho_2(T)\sqrt{T}\,\|g_{xx}(x,t)\|_{L_2(D_T)}\,,$$
$$A_2(T) = \left\|[h(t)]^{-1}\right\|_{C[0,T]} \times$$
$$\times\left\{\left\|\left(h_1'''(t) + h_1''(t) - \int_0^1 \omega(x) f(x,t)\, dx\right) g(0,t) -\right.\right.$$
$$\left.-\left(h_2'''(t) + h_2''(t) - f(0,t)\right)\int_0^1 \omega(x) g(x,t)\, dx\right\|_{C[0,T]} +$$
$$+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|g(0,t)\|_{C[0,T]}\,\|\omega(x)\|_{L_2(0,1)} + \left\|\int_0^1 \omega(x) g(x,t)\, dx)\right\|_{C[0,T]}\right) \times$$
$$\times\left(\rho_3(T)\left\|\varphi_0'''(x)\right\|_{L_2(0,1)} + \rho_4(T)\left\|\varphi_1'''(x)\right\|_{L_2(0,1)} +\right.$$
$$\left.\left.+\rho_5(T)\left\|\varphi_2''(x)\right\|_{L_2(0,1)} + \rho_5(T)\sqrt{T}\,\|f_{xx}(x,t)\|_{L_2(D_T)}\right)\right\},$$
$$B_2(T) = \left\|[h(t)]^{-1}\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2} \times$$

$$\times\left(\|g(0,t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}+\left\|\int_0^1\omega(x)g(x,t)\,dx\right\|_{C[0,T]}\right)\rho_5(T)T\ ,$$

$$C_2(T)=\left\|[h(t)]^{-1}\right\|_{C[0,T]}\times$$

$$\times\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|g(0,t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}+\right.$$

$$\left.+\left\|\int_0^1\omega(x)g(x,t)\,dx\right\|_{C[0,T]}\right)\rho_5(T)\sqrt{T}\,\|g_{xx}(x,t)\|_{L_2(D_T)}\ ,$$

$$A_3(T)\ =\ \left\|[h(t)]^{-1}\right\|_{C[0,T]}\times\left\{\left\|\left(h_2'''(t)+\ h_2''(t)\ -f(0,t)\right)\ h_1(t)\ -\right.\right.$$

$$-\left.\left(h_1'''(t)+\ h_1''(t)\ -f(0,t)-\int_0^1\omega(x)f(x,t)\,dx\right)h_2(t)\ \right\|_{C[0,T]}+$$

$$+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|h_1(t)\|_{C[0,T]}\ +\|h_2(t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}\right)\times$$

$$\times\left(\rho_3(T)\left\|\varphi_0'''(x)\right\|_{L_2(0,1)}+\rho_4(T)\left\|\varphi_1'''(x)\right\|_{L_2(0,1)}+\right.$$

$$\left.\left.\rho_5(T)\left\|\varphi_2''(x)\right\|_{L_2(0,1)}+\rho_5(T)\sqrt{T}\,\|f_{xx}(x,t)\|_{L_2(D_T)}\right)\right\}\ ,$$

$$B_3(T)=\left\|[h(t)]^{-1}\right\|_{C[0,T]}$$

$$\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|h_1(t)\|_{C[0,T]}\ +\|h_2(t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}\right)\rho_5(T)T\ ,$$

$$C_3(T)=\left\|[h(t)]^{-1}\right\|_{C[0,T]}\times$$

$$\times\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{1/2}\left(\|h_1(t)\|_{C[0,T]}\ +\|h_2(t)\|_{C[0,T]}\|\omega(x)\|_{L_2(0,1)}\right)$$

$$\rho_5(T)\sqrt{T}\,\|g_{xx}(x,t)\|_{L_2(D_T)}\ .$$

From inequalities (28) - (30) we conclude:

$$\|\tilde{u}(x,t)\|_{B_{2,T}^3}+\|\tilde{a}(t)\|_{C[0,T]}+\left\|\tilde{b}(t)\right\|_{C[0,T]}\leq$$

$$\leq\ A(T)+B(T)\,\|a(t)\|_{C[0,T]}\,\|u(x,t)\|_{B_{2,T}^{3,}}+C(T)\,\|b(t)\|_{C[0,T]}\,,\tag{31}$$

where

$$A(T)=A_1(T)+A_2(T)\ +A_3(T)$$

$$B(T)=B_1(T)+B_2(T)\ +B_3(T)\,,C(T)=C_1(T)+C_2(T)\ +C_3(T)\,.$$

So, the following theorem is proved.

Theorem 2. *Let conditions 1-5 be satisfied and*

$$(A(T)+2)(B(T)(A(T)+2)+C(T))<1\,. \tag{32}$$

Then the problem (1)-(3), (6),(7) *has a unique solution in the ball* $K=K_R(||\,z\,||_{E_T^3}\leq R=A(T)+2)$ *from* E_T^3.

Proof. In space E_T^3 consider the equation

$$z=\Phi z\,, \tag{33}$$

where $z=\{u,\,a,b\}$, the components $\Phi_i(u,a,b)$ $(i=1,2,3)$, of the operator $\Phi(u,a,b)$, are defined by the right-hand sides of equations (14), (21), (22).

Consider the operator $\Phi(u,a,b)$ in the ball $K=K_R$ from E_T^3. Similarly to (31), we obtain that for any $z_1,z_2,z_3\in K_R$ the following estimates are valid:

$$\|\Phi z\|_{E_T^3}\leq A(T)+B(T)\,\|a(t)\|_{C[0,T]}\,\|u(x,t)\|_{B_{2,T}^3}+C(T)\,\|b(t)\|_{C[0,T]}\,,$$
$$\leq A(T)+(A(T)+2)(B(T)(A(T)+2)+C(T)), \tag{34}$$

$$\|\Phi z_1-\Phi z_2\|_{E_T^3}\leq$$
$$\leq B(T)R(\|a_1(t)-a_2(t)\|_{C[0,T]}+\|u_1(x,t)-u_2(x,t)\|_{B_{2,T}^3})+C(T)\,\|b_1(t)-b_2(t)\|_{C[0,T]}\,. \tag{35}$$

Then, by virtue of (32), from (34) and (35), it is clear that the operator $\Phi(u,a,b)$, satisfies the conditions of the contraction mapping principle on the set $K=K_R$. Therefore, the operator $\Phi(u,a,b)$, in the ball $K=K_R$ has a unique fixed point $\{z\}=\{u\,,\,a,b\}$, which is a solution to equation. (33), i.e. is the only solution of systems (14), (21), (22) in the ball $K=K_R$.

The function $u(x,t)$, as an element of space $B_{2,T}^3$, is continuous and has continuous derivatives $u_x(x,t)$, $u_{xx}(x,t)$ in D_T.

Similarly, [7], it can be shown that $u_t(x,t)$, $u_{txx}(x,t)$ $u_{tt}(x,t)$, $u_{ttt}(x,t)$ are continuous in D_T.

It is easy to check that equation (1), conditions (2), (3), (6) and (7) are satisfied in the usual sense. Then, $\{u(x,t),a(t),b(t)\}$ is a solution of problem (1) - (3), (6), (7). By the corollary of Lemma 1, it is unique in the ball $K=K_R$. Theorem is proved.

Using Theorem 1, the last theorem implies the unique solvability of the initial problem (1) - (4).

Theorem 3. *Let all conditions of Theorem 2 be satisfied and*

$$\int_0^1\omega(x)\varphi_0\,(x)\,dx=h_1\,(0)\,,\ \int_0^1\omega(x)\varphi_1\,(x)\,dx=h_1'\,(0)\,,\ \int_0^1\omega(x)\varphi_2\,(x)\,dx=h_1''\,(0)\ ,$$
$$\varphi_0\,(0)=h_2\,(0)\,,\ \ \varphi_1\,(0)=h_2'\,(0)\ ,\ \ \varphi_2\,(0)=h_2''\,(0)\ ..$$

Then problem (1) - (5) has a unique classical solution in the ball $K=K_R(||\,z\,||_{E_T^3}\leq R=A(T)+2)$ *from* E_T^3 .

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