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Integral Inequalities for Function Spaces with a Finite Collection of Generalized Smoothness

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Abstract. In this paper the function space $\bigcap\limits_{k=0}^{n} \Lambda_{p_k,\theta_k}^{\langle m^k;N^k\rangle}(G,\varphi_k)$ is defined. This function spaces is the generalization of classical Sobolev-Slobodetskii and Nikolskii-Besov spaces. We established sufficient conditions under which the embedding theorems for these spaces are proved. We reduce the analog of integral representations of functions given by S.L. Sobolev for functions form the space $\bigcap\limits_{k=0}^{n} \Lambda_{p_k,\theta_k}^{\langle m^k;N^k\rangle}(G,\varphi_k)$.

Key Words and Phrases: **Key Words and Phrases:** Generalized Hölder space, strong $a(h)$-horn condition, integral representation, embedding theorem.

2010 Mathematics Subject Classifications: 46E30, 46E35

1. Introduction

The theory of embedding of spaces of differentiable functions of several variables developed as a new direction of mathematics in the 30s of the 20th century as a result of the works of S.L. Sobolev, which is presented in detail in monograph [5]. This theory studies important connections and relations of differential properties of functions in various metrics. In addition to its independent interest from the point of view of function theory, it also has numerous and effective applications in the theory of partial differential equations. Such applications were given by S.L. Sobolev in [5] (see, also [3]). S.L. Sobolev studied isotropic spaces $W_p^{(l)}(G)$ of functions f defined on a domain $G \subset R^n$ with the norm

$$\|f\|_{W_p^{(l)}(G)} = \sum_{|\alpha|\le l} \|D^\alpha f\|_{L_p(G)} ,$$

where l is a non-negative integer and $p \ge 1$. S.L. Sobolev proved embedding theorems for function space $W_p^{(l)}(G)$ in domains of n-dimensional Euclidean spaces. Namely, theorems on the summability in power q of derivatives $D^\beta f$ with respect to a domain G or manifolds of lower dimension belonging to G.

In subsequent years, the theory of embedding developed intensively in various directions and received new interesting and important applications. Among these works, one can note the works of S.M. Nikolskii, O.V. Besov, V.P. Ilin, N. Aronszajn, V.M. Babich, L.N. Slobodetskii, A.S. Jafarov, G. Freud, D. Kralik, V.I. Burenkov, A.J. Jabrailov and others. For more details we refer the readers to [1] and [4].

S.L. Sobolev established embedding theorems using integral representations of functions in terms of their weak derivatives. This method of integral representations was developed in the works of V.P. Ilin and, in particular, was carried over to cases of representation through differences. One of the significant advantages of the method of integral representations is that the representation of a function at a given point is constructed from the values of this function at the points of a bounded cone with vertex at this point. This creates an opportunity to study function spaces of functions defined on an open set of a sufficiently general form.

The remainder of the paper is structured as follows. Section 2 contains some preliminaries along with the standard ingredients used in the proofs. In Section 3 we reduce the class of domains satisfying special horn conditions. Our principal assertions, concerning the embedding of Hölder spaces with generalized smoothness to Lebesgue spaces are formulated and proved in Section 4. We establish sufficient conditions on a domain $G \subset R^n$ for the validity of embedding theorem.

2. Preliminaries

Let R^n be the $n-$ dimensional Euclidean space of points $x = (x_1, ..., x_n)$, and let G be a Lebesgue measurable set of R^n. Suppose $f : G \to R^n$ is a Lebesgue measurable function and let $1 \leq p < \infty$. Throughout this paper we will assume that all sets and functions are Lebesgue measurable.

Definition 1. *The Lebesgue space $L_p(G)$ is the class of all measurable functions f defined on G such that*

$$\|f\|_{L_{p(G)}} = \|f\|_{p,G} = \left(\int_G |f(x)|^p \, dx \right)^{\frac{1}{p}}. \tag{1}$$

In the case $p = \infty$, the space $L_\infty(G)$ will be defined as all measurable functions such that

$$\|f\|_{\infty,G} = vrai \sup_{x \in G} |f(x)|. \tag{2}$$

Let

$$\left. \begin{array}{c} m = (m_1, .., m_n) \\ N = (N_1, .., N_n) \end{array} \right\} \tag{3}$$

be the vectors with integer non-negative components.

The mixed derivative of order $|m| = m_1 + ...m_n$ is defined by

$$D^m f(x) = Dl_1{}^{m_1} ... D_n{}^{m_n} f(x_1, .., x_n) = \frac{\partial^{|m|}}{\partial x_1{}^{m_1} ... \partial x_n{}^{m_n}}. \tag{4}$$

We denote by

$$\Delta^N(t) f(x) = {\Delta_1}^{N_1}(t_1) \ldots {\Delta_n}^{N_n}(t_n) f(x_1, .., x_n) \tag{5}$$

the $|N| = N_1 + \ldots N_n$-order finite mixed difference of a function $f = f(x)$, corresponding to mixed step of a vector $t = (t_1, ..t_n)$. Here

$$\begin{cases} {\Delta_k}^{N_k}(t_k) f(\ldots x_k \ldots) = {\Delta_k}^1(t_k) \left\{ {\Delta_k}^{N_k - 1}(t_k) f(\ldots x_k \ldots) \right\} \\ {\Delta_k}^0(t_k) f(\ldots, x_k, \ldots) = f(\ldots, x_k, \ldots) \\ {\Delta_k}^1(t_k) f(\ldots, x_k, \ldots) = f(\ldots, x_k + t_k, \ldots) - f(\ldots, x_k, \ldots) \end{cases} \tag{6}$$

Therefore

$${\Delta_k}^{N_k}(t_k) f(\ldots, x_k, \ldots) \tag{7}$$

Is the finite difference of a function $f = f(x)$ of order N_k in the direction of variable x_k with step t_k.We observe that in the domain $G \subset E_n$ the expression

$$\Delta^N(t, G) f(x) = \Delta^N(t) f(x) \, \Delta^N(t, G) f(x) = \Delta^N(t) f(x), \tag{8}$$

is the mixed difference of a function $f = f(x)$. In this case we suppose that the mixed difference is constructed from the vertices of a polyhedron that lies entirely in the domain $G \subset E_n$. Otherwise, we assume that

$$\Delta^N(t, G) f(x) = 0. \tag{9}$$

Let $\varphi = \varphi(t) = (\varphi_1(t_1), \ldots, \varphi_n(t_k))$ be a vector-function such that $\varphi_j = \varphi_j(t_j) > 0$, if $t_j \neq 0$ and $\varphi_j(t_j) \downarrow 0$ for $t \to 0$ and for all $j = 1, 2, .., n$.

Let $1 \le \theta < \infty$ and let $\frac{dt}{t} = \prod_{j \in E_n} \frac{dt_j}{t_j}$. We consider the following semi-norm

$$\|f\|_{\Lambda_{p,\theta}^{\langle m,n \rangle}}(G, \varphi) = \left\{ \int_{E_{|E_n|}} \left\| \frac{\Delta^N\left(\frac{t}{N}; G\right) D^m f}{\prod_{j \in E_n} \varphi_j(t_j)} \right\|_{p,G}^{\theta} \frac{dt}{t} \right\}^{\frac{1}{\theta}}. \tag{10}$$

For $\theta = \infty$, we suppose that

$$\|f\|_{\Lambda_{p,\infty}^{\langle m,n \rangle}(G,\varphi)} = vrai \sup_{t \in E_N} \left\| \frac{D^N\left(\frac{t}{N}, G\right)}{\prod_{j \in E_n} \varphi_j(t_j)} \right\|_{P,G}. \tag{11}$$

Here $E_n = \sup p \; N$ is a support of a vector $N = (N_1, \ldots N_n)$. In other words E_n is a set of nonzero indices of the coordinates of vector N. Thus, $E_n \subset \{1, 2, \ldots n\} = e_n$.

Let us $\frac{t}{N} = \left(\frac{t_1}{N_1}, \ldots \frac{t_n}{N_n} \right)$ and we use the convention $\frac{0}{0} = 0$.

Therefore $E_{|E_N|} = \left\{ t \in E_N; t_j = 0 \left(j \in {}^{e_n}/_{E_n} \right) \right\}$.

Let

$$\left. \begin{array}{l} m^k = \left(m_1^k, .. \, m_n^k\right) \\ N^k = \left(N_1^k, \ldots, N_n^k\right) \end{array} \right\} \qquad (k = 0, 1, \ldots, n) \tag{12}$$

be the vectors with integer non-negative components. Thus,

$\left. \begin{array}{r} m_j^k \ge 0 \\ N_j^k \ge 0 \end{array} \right\}$ for all $\left(j = \overline{1, n}\right)$ and $k = \overline{0, n}$.

Suppose that any vector-function from collection of $(n+1)$ vector function $\varphi^k = \varphi^k(t) = \left(\varphi_1^k(t_1, ..., \varphi_n^k(t_n))\right)$ satisfy following conditions:
$\varphi_j = \varphi_j(t_j) > 0$ for $t_j \neq 0$
$\varphi_j(t_j \downarrow 0)$ for $t \to 0$.

Definition 2. *Let* $1 \leq p_k \leq \theta_k \leq \infty$, *and* $k = 0, ..., n$. *The space*

$$\bigcap_{k=0}^{n} \Lambda_{p_k,\theta_k}^{\langle m^k;N^k\rangle}(G,\varphi_k) \tag{13}$$

is defined as the closure of sufficiently smooth functions $f = f(x)$ *with compact support on* R^n *by the norm*

$$\|f\|_{\bigcap\limits_{k=0}^{n} \Lambda_{p_k\theta_k}^{\langle m^k;N^k\rangle}(G;\varphi^k)} = \sum_{k=0}^{n} \|f\|_{\Lambda_{p_k\theta_k}^{\langle m^k,N^k\rangle}(G,\varphi_k)} < \infty. \tag{14}$$

Remark 1. *We observe that the space given by* (13) *in the case* $1 \leq p_k \leq \theta_k \leq \infty$ *(* $k = 0, n$ *) is a generalization of the classical Sobolev-Slobodetskii space* $W_p^r(G)$. *Also, in the case* $1 \leq p_k \leq \theta_k \leq \infty$ *and* $\sup p m^k \subseteq \sup p N_k = E_{N^k}$ *the space* $\bigcap\limits_{k=0}^{n} \Lambda_{p_k,\theta_k}^{\langle m^k;N^k\rangle}(G,\varphi_k)$ *is a generalization of Nikolskii-Besov space* $B_{p,\theta}^r(G)$ *(see, [2]).*

3. The class of domains $G \subset E_n$

Let

$$a(\upsilon) = (a_1(\upsilon), ..., a_n(\upsilon))\, a(\upsilon) = (a_1(\upsilon), ..., a_n(\upsilon)) \tag{15}$$

be a differentiable vector-function in $[0;h]$ such that

$$\left.\begin{array}{ll} a_j = a_j(\nu) > 0\,, & \upsilon \in (0;h] \\ \lim\limits_{\upsilon\to 0+} a_j(\upsilon) = 0 & \\ \frac{d}{d\upsilon} a_j(\upsilon) > 0\,, & \upsilon \in (0;h] \end{array}\right\} \tag{16}$$

for all $j = 1, n$.
Let $\delta = (\delta_1, ..., \delta_n)$ be a vector such that $\delta_j = \pm 1$. We put
$R_\delta(a(h)) = \bigcup\limits_{0<\upsilon\leq h} \left\{y \in E_n; c_j \leq \frac{y_j - \delta_j}{a_j(\upsilon)} \leq A_j^*\right\} \left(j = \overline{1,n}\right)$ for all $\upsilon \in (0;h]$. (**??**)
The set $X + R_\delta(a(h))$ is called $\leq a(h) \geq$-horn with vertices in $x \in R^n$.

Note that at each point $x \in R^n$, you can give2^n-number of "$a(h)$"-horns with vertex in $x \in R^n$.

If a vector $\delta = (\delta_1, ..., \delta_n)$ be fixed, then at each point $x \in R^n$ there is only single $\leq a(h) \geq$-horn (for the same vector function (15) - (16) the vertex at this point $x \in R^n$).

A subdomain $\Omega \subset G$ is considered to be a subdomain satisfying the $a(h)$-horn condition, if there is a vector $\delta = (\delta_1, ..., \delta_n)$ with $\delta_j = \pm 1$ for which $X + R_\delta(a(h)) \subset G$ for all $x \in \Omega$.

Definition 3. *A subdomain $G \subset E_n$ is called a domain satisfying "$a(h)$- horn" condition, if there exists a finite collection of subdomains*

$$\Omega_1, \Omega_2, ..., \Omega_m \subset G,$$

with $a(h)$-horn condition such that $\bigcup\limits_{k=1}^{M} \Omega_k = G$.

By $C(a(h))$ we denote the class of domains $G \subset E_n$ satisfying the $a(h)$-horn condition.

Definition 4. *Let $k = \overline{1, M}$ and let $\Omega_{k,\varepsilon} = \{y \in \Omega_k; \rho(y; G \backslash \Omega_k) > \varepsilon\}$ is a set of points $y \in \Omega_k$ spaced from $G \backslash \Omega_k$ at a distance greater than $\varepsilon > 0$. A set $G \in C(a(h))$ is called a domain satisfying strong $a(h)$-horn condition, if in addition to condition $\bigcup\limits_{k=1}^{M} \Omega_k = C$, there is also a covering*

$\bigcup\limits_{k=1}^{M} \Omega_{k,\varepsilon} \supseteq G$ *for some* $\varepsilon > 0$.

By $C_\varepsilon(a(h))$ we denote the class of domains $G \subset R^n$ satisfying strong "$a(h)$ -horn".

We observe that the notions of a domain $G \subset R^n$ satisfying the $a(h)$-horn condition and strong $a(h)$-horn conditions are introduced in [3] by O.V. Besov, respectively.

4. Main results

In this section of our paper we state and prove our principal assertions.

Theorem 1. *Let $1 \le p_k \le \theta_k \le \infty$ and let $f \in \bigcap\limits_{k=0}^{n} \Lambda_{p_k,\theta_k}^{\langle m^k; N^k \rangle}(G, \varphi^k)$ $\left(k = \overline{0, n}\right)$. Suppose that $\left.\begin{array}{l} m^k = \left(m_1^k, ..., m_n^k\right) \\ N^k = \left(N_1^k, ..., N_n^k\right) \end{array}\right\}$ is the vectors with integer non-negative components such that $\{k\} \subset \sup p\left(m^k + N^k\right)$ $\qquad (k = 1, \overline{n})$.*

Let $\varphi^k(t) = \left(\varphi_1^k(t_1), ..., \varphi_n^k(t_n)\right)$ be a vector-function satisfying condition $\varphi_j(t) = \varphi_j(t_j) > 0$ for $t_j \ne 0$, and $\varphi_j(t_j) \downarrow 0$ for $t \to 0$. Suppose that a domain $G \subset E_n$ is satisfy "$a(h)$ -horn" condition, i.e. $G \in C(a(h))$ and a vector-function $a(\upsilon) = (a_1(\upsilon), ..., a_n(\upsilon))$ satisfy condition (16) for all $\upsilon \in [0, n)$

Let $\upsilon = (\upsilon_1, ..., \upsilon_n)$ be a vector with integer-nonnegative components satisfy matching condition with respect to the vectors $\left.\begin{array}{l} m^k = \left(m_1^k, ..., m_n^k\right) \\ N^k = \left(N_1^k, ..., N_n^k\right) \end{array}\right\}$ $m_j^k \ge 0, \quad N_j^k \ge 0$, as the form:

$$\left.\begin{array}{ll} \nu_j \ge m_j^0 + N_j^0 & (j = 1, 0) \\ \nu_j \ge m_j^k + N_j^k & (j \ne k) \\ \nu_k < m_k^k + N_k^k & (j = k) \end{array}\right\}, (k = 1, n) .$$

Here $H_k(h) \le const < \infty$, $\qquad 1 \le p_k \le q < \infty \qquad \left(k = \overline{1, n}\right)$ and

$$H_k = \int_0^h \prod_{j=1}^{n} (a_j(\upsilon))^{m_j^k - \frac{\upsilon}{j} - \frac{1}{p_k} - \frac{1}{q}} \left\{ \prod_{j \in E_{N_k}} \varphi_j^k(a_j(\upsilon)) \right\} \frac{da_k(\upsilon)}{a_k(\upsilon)} \tag{17}$$

for all $k \in e_n = \{1, 2, ..., n\}$.

Then

$$D^{\upsilon} f \in L_q(G), \tag{18}$$

and the integral inequality holds

$$\|D^{\upsilon} f\|_{L_q(G)} \leq C \sum_{k=0}^{n} H_k(h) \|f\|_{\Lambda^{\langle m^k, n^k \rangle}_{p_k, \theta_k}(G; \varphi^k)}, \tag{19}$$

where $C > 0$ is a constant independent of function $f = f(x)$ and $h > 0$. Also $H_k(h)$ is defined by (17) for $k = \overline{1, n}$, and for $k = 0$

$$H_0(h) = \prod_{j=1}^{n} (a_j(h))^{m_j^o - \upsilon_j - \frac{1}{p_0} + \frac{1}{q}} \prod_{j \in E_{N_0}} \varphi_j^0(a_j(h)).$$

We observe that $E_{N_k} = \sup p N^k \quad \left(k = \overline{0, n}\right)$.
Other formulation of Theorem 3.1 we can give as following form.

Remark 2. *Under the conditions of Theorem 3.1 the following embedding holds:*

$$D^{\nu} : \bigcap_{k=0}^{n} \Lambda^{\langle m^k, N^k \rangle}_{p_k, \theta_k}\left(G; \varphi^k\right) \subset L_q(G) \tag{20}$$

In particular, for $\upsilon = 0$ *we have* $\bigcap_{k=0}^{n} \Lambda^{\langle m^k, N^k \rangle}_{p_k, \theta_k}\left(G; \varphi^k\right) \subset L_q(G)$.

Thus, the inclusion (20) *is characterized the differential properties of functions from* $\bigcap_{k=0}^{n} \Lambda^{\langle m^k, N^k \rangle}_{p_k, \theta_k}\left(G; \varphi^k\right)$.

Proof. Theorem 3.1 is proved by the method of integral representations of functions $f = f(x)$, developed by S.L. Sobolev in [1]. The method of the proof of Theorem 3.1 is the integral identities given by the equality

$$D^{\nu} f = (-1)^{|\nu + m^0|} C_0 A_0(h) \int_{E_{|E_{N^0}|}} dz^0 \times \int_{E_n} \left\{ \Delta^{N^0} \left(\frac{Z^0}{N^0}\right) D^{m^0} f(x+y) \right\} M_{\delta,0} dy +$$

$$+ \sum_{k=1}^{n} (-1)^{|\nu + m_k|} C_k \int_0^h A_k(\upsilon) \frac{d a_k(\upsilon)}{a_k(\upsilon)} \times$$

$$\times \int_{E_{|E_{N^k}|}} dz^k \int_{E_n} \left\{ \Delta^{N^k} \left(\frac{z^k}{N^k} D^{m^k} f(x+y) \right) \right\} M_{\delta,k} dy. \tag{21}$$

Here C_k are the constants independent on $f = f(x)$ and $h > 0$, where

$$\left|\nu + m^k\right| = \sum_{j=0}^{n} \left(\nu_j + m_j^k\right),$$

$$A_k(\upsilon) = \prod_{j=1}^{n} (a_j(\upsilon))^{m\frac{k}{j}-\nu_j-1} \prod_{j\in E_{N^k}} (a_j(\upsilon))^{-1} \qquad \left(k=\overline{0,n}\right).$$

In (21), the kernels $M_{\delta,0}$ and $M_{\delta,k}$

$$M_{\delta,0} = M_{\delta,0}\left(\frac{y}{a(h)}; \frac{z^0}{a(h)}\right),$$

$$M_{\delta,k} = M_{\delta,k}\left(\frac{y}{a(h)}; \frac{z^k}{a(h)}\right), (k=\overline{1,n})$$

are sufficiently smooth functions with compact support on R^n, respectively. Here

$$\frac{y}{a(\upsilon)} = \left(\frac{y_1}{a_1(\upsilon)}, ..., \frac{y_n}{a_n(\upsilon)}\right),$$

$$\frac{z^k}{a(\upsilon)} = \left(\frac{z_{k,1}}{a_1(\upsilon)}, ..., \frac{z_{k,n}}{a_n(\upsilon)}\right), k=(1,n)$$

while the supports of these kernels satisfy condition:

$$\sup p M_{\delta,k}\left(y; z^k\right) \subset \left\{ \begin{array}{l} 0 < y_j - \delta_j \le 1 \qquad (j=\overline{1,n}) \\ \left(y; z^k\right) \in E_n \times E_{\left|E_{n^k}\right|} : 0 < z_{k,j}\delta_j \le 1 \end{array} \right\}, (j \in E_{N_k}).$$

Also, a vector $\delta = (\delta_1, ..., \delta_n)$, with $\delta_j = \pm 1$ $\left(j=\overline{1,n}\right)$ be fixed.

We observe that in (22)

$$\int_{E_{\left|E_{N^k}\right|}} (...)\, dz^k = \underbrace{\int_{E_1} .... \int_{E_1}}_{E_{N^k}} (....) \prod_{j\in E_{N^k}} dz_{k,j},$$

where by $|E_{N_k}|$ the number of elements of the set $E_{N_k} = \sup p N^k$.
Moreover, the construction of auxiliary functions given by equality

$$f_{\nu,\delta^i}(x) = (-1)^{\left|m^0+\nu\right|} C_0 A_0(h) \int_{\left|E^{N^0}\right|} dz^0 \times$$

$$\times \int_{E_n} \left\{\Delta^{N^0}\left(\tfrac{z^0}{N^0}; \Omega_i + R_{\delta^i}\right) D^{m^0} f(x+y)\right\} M_{\delta^i 0}\, dy +$$

$$+ \sum_{k=1}^{n} (-1)^{\left|m^k+\nu\right|} C_k \int_0^h A_k(\upsilon) \frac{da_k(\upsilon)}{a_k(\upsilon)} \int dz^k \times$$

$$\int_{E_n} \left\{\Delta^{N^k}\left(\frac{z^k}{N^k}; \Omega_i + R_{\delta^i}\right) D^{m^k} f(x+y)\right\} M_{\delta^i,k}\, dy = J_{0,\delta^i}(f) + \sum_{k=1}^{n} J_{0,\delta^i}(f) \qquad (i=1,n) \tag{22}$$

and proof of inequality

$$\|D^{\nu}f\|_{q,G} \leq C\sum_{i=1}^{M}\|D^{\nu}f\|_{q,\Omega_i+R_{\delta_i}} \leq C\sum_{i=1}^{M}\|f_{\nu,\delta_i}\|_{q_1E_k} \leq C\sum_{i=1}^{M}\sum_{k=0}^{n}\left\|J_{k,\delta^i}\right\|_{q_1E_n} \tag{23}$$

shows that estimates of integral expressions $\|D^{\nu}f\|_{q,G}$ reduce to estimates of integral operators J_{k,δ^i} $\left(k=\overline{0,n}\right)$ in Lebesgue space $L_q(G)$.
Then, using the Hölder inequality and Young inequality for convolution, we have (see, [2])

$$\|D^{\nu}f\|_{q,G} \leq C\sum_{i=1}^{M}\sum_{k=0}^{n}\left\|J_{k,\delta^i}(f)\right\|_{q_1E_n} \leq C\sum_{k=0}^{n}Q_k(h)\left(\sum_{i=1}^{M}\|f\|\,\Lambda_{p_k,\theta_k}^{\langle m_k,N_k\rangle}\left(\Omega_i+R_{\delta_i},\varphi_k\right) \leq\right.$$

$$\leq C\sum_{k=0}^{n}Q_k(h)\,\|f\|_{\Lambda_{p_k,\theta_k}^{\langle m_k,N_k\rangle}(G;\varphi_k)}\,.$$

Here

$$Q_k(h)=\int_0^h\prod_{j=1}^{n}\left((a_j(\upsilon))^{m\frac{k}{j}-\nu_j-1}\right)\left\{\prod_{j\in E_{N^k}}\varphi_j^k(a_j(\upsilon))\right\}\frac{da_k(\upsilon)}{a_k(\upsilon)}\,.$$

This complete the proof of Theorem 3.1.

References

[1] O.V. Besov, S.M. Nikolskii, V.P. Ilin, Integral representation of functions and embedding theorems, M.Nauka, 1975. (in Russian)

[2] A.J. Jabrailov, N.I. Guliyev, Intermediate embedding theorems for function spaces with generalized smoothness. Rep. Acad. Sci., 45(**23**), 1989, 49-53.

[3] A.M. Najafov, A.T. Orujova, On the solution of a class of partial differential equations. Electron. J. Qual. Theory Differ. Equ., 2017(**44**), 2017, 1-9.

[4] S.M. Nikolskii. Approximation of functions of several variable and imbedding theorems. Springer-Verlag, Berlin-Heidelberg-New-York, 1975.

[5] S.L. Sobolev, Some applications of functional analysis in mathematical physics. Nauka, Moscow, 1988. (in Russian)

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