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Motion of Gas-Liquid Mixture in a Connected System Reservoir- Pipeline

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Abstract. An integral mathematical model of the process of unsteady motion of a gas-liquid mixture in the reservoir-pipeline system when connecting the main line of new sources has been built and the associated equations have been solved. An analytical formula has been obtained that makes it possible to determine the dynamics of pressure at the bottom hole, wellhead and reservoir productivity depending on the parameters of the system. Numerical calculations are presented for various values of the system parameters.

Key Words and Phrases: **Key Words and Phrases:** model, coupled system, differential equations, filtration, Laplace transform, original, liquid.

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1. Introduction

Determination of pressure at the bottom of the well using wellhead information for gas and gas condensate fields with abnormally high reservoir pressure is of great scientific and practical importance. The works [1-5,9,10] are devoted to this problem. In some of these works [1-5], the pressure at the bottom of the well is determined for the stationary case of motion without taking into account the dynamic connection of the reservoir-well system. And in others [9, 10], the filtration process is simplified or the problem is solved by numerical methods. In works [11-13] methods of conjugation of models for reservoir and well are presented.

The work [15] is devoted to the physical and mathematical formalization, development and software implementation of computational algorithms for modeling unsteady three-phase flows in the conjugated reservoir-well ESP system, which significantly differs from the considered model of the filtration process and the flow of a gas-liquid mixture in the conjugated reservoir-pipeline system.

Investigation of the filtration of a gas-liquid mixture and aerated liquid in one-dimensional and two-dimensional models are devoted to works [16-20].

In these works, studies are carried out without taking into account the filtration process and fluid flow in the conjugated reservoir-well system.

n reality, the process of filtration and movement of liquid in the riser pipes and in the main line is interconnected. Therefore, the filtration and movement of liquid in the riser pipes and in the main line must be considered together, which is what this work is devoted to.

In [8], the influence of connecting a new source to an existing oil pipeline on the operating mode of oil wells is determined. And in most cases, not pure oil flows through the main line, but a gas-liquid mixture. Therefore, determining the impact of connecting a new source to the existing trunk line on the operating mode of gas and gas condensate wells is also of great scientific and practical importance.

A rigorous solution to this problem is to take into account the interaction in the reservoir-well system. At the same time, it is necessary to consider and investigate the system of equations describing the joint flow of the gas-liquid mixture in the reservoir and the wellbore and in the main line. These are nonlinear differential equations, and it is not possible to obtain their exact analytical solution. Therefore, the analytical solution of the problem posed can be carried out approximately with an accuracy sufficient for practice [6, 20].

Formulation of the problem. Let the reservoir have a circular shape with a radius R_k. The outer boundary of the formation is impermeable. The well with radius r_c is located concentrically to the outer boundary of the reservoir. The reservoir is assumed to be homogeneous in terms of permeability. It is assumed that at the initial moment the formation is filled with a gas-liquid mixture with a low concentration of liquid. The well is in operation. During the production of a gas-liquid mixture, a pressure drop occurs at all points of the formation, as well as at the wellhead.

The mass G_0 of the gas-liquid mixture in the formation at each moment of time can be determined by the formula

$$G_0 = 2\pi hm \int_{r_c}^{R_k} \rho_{cm} r dr. \tag{1}$$

The initial and boundary conditions required for the filtration of a gas-liquid mixture are as follows:

$$P|_{t=0} = P_{\mathrm{k}}, r = R_k, \tag{2}$$

$$P|_{r=r_c} = P_{\mathrm{c}}(t), t > 0, \tag{3}$$

$$\left.\frac{\partial P}{\partial r}\right|_{r=R_k} = 0\,, t > 0 \tag{4}$$

The density of the mixture can be determined from the formula [14]:

$$\frac{1+\eta}{\rho_{cm}} = \frac{1}{\rho_o} + \frac{\eta}{\rho_q}$$

$$\rho_{cm} = \frac{(1+\eta)\rho_o\rho_q}{\rho_o + \eta\rho_q} \tag{5}$$

If the process is taken to be isothermal, then the gas density will be

$$\rho_q = P\frac{\rho_{atm}}{P_{atm}} \tag{6}$$

Substituting expression (6) into equation (5), we obtain

$$\rho_{cm} = \frac{1+\eta}{\eta}\rho_o x \frac{1}{1+x} \tag{7}$$

where

$$x = \frac{\eta \rho_{atm}}{\rho_o P_{atm}} P$$

$x << 1$ for practical values of the parameters always.
Therefore, $\frac{1}{1+x}$ can be represented as

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... \tag{8}$$

In the first approximation, taking into account only one term of series (8), from expression (7) we obtain

$$\rho_{cm} = \frac{(1+\eta)\rho_{atm}}{P_{atm}} P \tag{9}$$

Then from expression (1) taking into account (9) we obtain

$$G_0 = 2\pi hm \frac{(1+\eta)\,\rho_{\text{atm}}}{P_{\text{atm}}} \int_{r_c}^{R_k} P \cdot r dr \tag{10}$$

Pressure P at any point in the reservoir will be sought in the form [6]

$$P = P_{\text{A}}(t) + A(t)\, f(r) \tag{11}$$

A function $f(r)$ satisfying the boundary conditions (3) and (4) will be sought in the form [6.14]

$$f(r) = \ln\frac{r}{r_c} - \frac{r}{R_k} + \frac{r_c}{R_k} \tag{12}$$

where $A(t)$ is unknown, depending on time t, function.
Substituting expressions (11) and (12) into expression (10), we obtain

$$G_0 = 2\pi hm \frac{(1+\eta)\,\rho_{\text{atm}}}{P_{\text{atm}}} \left(P_c(t)\, \frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} D A(t) \right) \tag{13}$$

where

$$D = \ln\frac{R_k}{r_c} - \frac{7}{6} + \frac{r_c}{R_k} + \frac{1}{2}\left(\frac{r_c}{R_k}\right)^2 - \frac{1}{3}\left(\frac{r_c}{R_k}\right)^3$$

The mass inflow of a gas-liquid mixture into a well per unit of time can be determined by the formula

$$G = -\frac{dG_0}{dt} \tag{14}$$

Then, substituting expression (13) into formula (14), we obtain

$$G = -2\pi hm\, \frac{(1+\eta)\ \rho_{\text{atm}}}{P_{\text{atm}}} \left(\dot{P}_c(t)\, \frac{R_k^2 - r_c^2}{2} + \frac{R_k^2}{2} D\, \dot{A}(t) \right) \tag{15}$$

On the other hand, the inflow of the gas-liquid mixture into the well per unit of time can be determined by the formula [14,23]

$$G = k\frac{P_c\left(T\right) + P_c(0)}{\mu\beta}\pi\, r_c h\ \left.\frac{\partial P}{\partial r}\right|_{r=r_c} \tag{16}$$

where $\beta = \frac{P_{atm}}{\rho_{atm}}$, μ_{cm}−mixture viscosity, k- formation permeability coefficient, $P_c(0)$,$P_c(T)$ - downhole pressure at the beginning and end of operation.
Substituting expressions (11) and (12) into formula (16) and equating the resulting expression with formula (15), we obtain a differential equation with respect to the variable$A\left(t\right)$:

$$\dot{A}\left(t\right) + \alpha A\left(t\right) = -\frac{1}{D}\dot{P}_c(t) \tag{17}$$

where $\alpha = k\frac{P_c(T)+P_c(0)}{\mu_{\mathrm{cm}} m {R_k}^2 D(1+\eta)}$
Integrating equation (17), we obtain

$$A(t) = A(0)\exp(-\alpha t) + \frac{1}{D}\int_0^t \dot{P}_c\left(\tau\right)\exp\left(-\alpha\left(t-\tau\right)\right)d\tau \tag{18}$$

where $A\left(0\right)$ is the constant of integration.
Substituting the resulting expression into formula (11), we obtain

$$P(r,t) = P_c(t) + (A(0)\exp(-\alpha t) + \frac{1}{D}\int_0^t \dot{P}_c\left(\tau\right)\exp\left(-\alpha\left(t-\tau\right)\right)d\tau)f(r) \tag{19}$$

From expression (19), taking into account the initial condition (2), we obtain

$$A(0) = \frac{P_k - P_c(0)}{\ln\frac{R_k}{r_c} - \frac{r_c}{R_k} - 1} \tag{20}$$

Now let's consider the movement of the gas-liquid mixture in the riser pipes.
Due to the smallness of η, let us take, in the first approximation, a gas-liquid mixture as a homogeneous gas. Then the equation of gas motion in the pipe and the equation of continuity are described by the equations of I.A. Charny [21, 22]:

$$-\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial t} + 2aQ + \rho_{cm}g \tag{21}$$

$$-\frac{\partial P}{\partial t} = c^2\frac{\partial Q}{\partial x}$$

where, $Q = \rho_{cm}u$, u averaged over the cross section of the flow velocity of the mixture in the column of lifting pipes, c- is the speed of sound propagation in the gas, t - is the time, x-is the coordinate, ρ_{cm}-is the density of the gas-liquid mixture, a-is the drag coefficient.
Substituting expressions (9) into the first equation of expression (21) and differentiating into the first equation of expression (21) in time x, and the second in t and subtracting one from the other, we get

$$\frac{\partial^2 P}{\partial t^2} = c^2\frac{\partial^2 P}{\partial x^2} - 2a\frac{\partial P}{\partial t} + \frac{1+\eta}{\beta}c^2 g\frac{\partial P}{\partial x} \tag{22}$$

At the initial moment, the movement of the mixture in the column of lifting pipes occurs in a stationary mode. Therefore, from expression (21) we will have

$$-\frac{\partial P}{\partial x} = 2aQ + \frac{(1+\eta)}{\beta} gP \tag{23}$$

C is the constant of integration.
Boundary conditions

$$P|_{x=0} = P_c(0) \tag{24}$$

Then, from expression (23), taking into account boundary condition (24) for the initial and boundary conditions of equation (22), we will have

$$P|_{t=0} = \frac{2a\beta Q_0}{(1+\eta)g}\left(\exp\left(-\frac{(1+\eta)}{\beta}gx\right) - 1\right) + P_c(0)\exp\left(-\frac{(1+\eta)}{\beta}gx\right) \tag{25}$$

$$\left.\frac{dP}{dt}\right|_{t=0} = 0 \tag{26}$$

$$P|_{x=0} = P_c(t) \tag{27}$$

$$P|_{x=l} = P_y(t) \tag{28}$$

The solutions of equation (22), taking into account the boundary conditions (27) and (28), will be sought in the form [7]

$$P = P_c(t) - \frac{P_c(t) - P_y(t)}{l}x + \sum_{i=1}^{n} \varphi_i(t)\sin\left(\frac{i\pi x}{l}\right) \tag{29}$$

where $\varphi_i(t)$ is an unknown time-dependent t function, l is the depth of descent pipes.
Substituting expression (29) into equation (22), multiplying both sides of the resulting expression by $\sin\left(\frac{i\pi x}{l}\right)$ and integrating it from 0 to l, we obtain the equation

$$\begin{aligned}&\ddot{\varphi}_i + 2a\dot{\varphi}_i + \tfrac{c^2 i^2\pi^2}{l^2}\varphi_i = -\tfrac{2}{\pi}\ddot{P}_c(t) - \tfrac{2}{\pi}\ddot{P}_y(t) + \tfrac{12a}{\pi}\dot{P}_A(t) - \tfrac{4a}{\pi}\dot{P}_y(t) + \\ &+ \tfrac{4(1+\eta)gc^2}{\beta l\pi}P_c(t) - \tfrac{4(1+\eta)gc^2}{\beta\pi l}P_y(t)\end{aligned} \tag{30}$$

Applying the Laplace transform, from equation (30), taking into account the initial conditions (25) and (26), we obtain

$$\begin{aligned}\bar{\varphi}_i = &\tfrac{s\varphi_i(0)}{(s-\xi_1)(s-\xi_2)} - \tfrac{2a\varphi_i(0)}{(s-\xi_1)(s-\xi_2)} + \tfrac{2sP_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \\ &+ \tfrac{2sP_y(0)}{\pi(s-\xi_1)(s-\xi_2)} - \tfrac{12aP_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \\ &+ \tfrac{4aP_y(0)}{\pi(s-\xi_1)(s-\xi_2)} - \tfrac{2s^2\bar{P}_c}{\pi(s-\xi_1)(s-\xi_2)} - \tfrac{2sP_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \\ &+ \tfrac{12as\bar{P}_c}{\pi(s-\xi_1)(s-\xi_2)} - \tfrac{4as\bar{P}_y}{\pi(s-\xi_1)(s-\xi_2)} + \\ &+ \tfrac{4(1+\eta)c^2 g\bar{P}_c}{\pi\beta l(s-\xi_1)(s-\xi_2)} - \tfrac{4(1+\eta)c^2 g\bar{P}_y}{\pi\beta l(s-\xi_1)(s-\xi_2)}\end{aligned} \tag{31}$$

where ξ_1 and ξ_2 are the roots of the equation

$$s^2 + 2as + \frac{c^2\pi^2 i^2}{l^2} = 0.$$

$\varphi_i(0)$ and $\dot{\varphi}_i(0)$ are determined from the initial conditions (25) and (26).
At the initial moment of time at $t = 0$, equating expressions (25) and (29).

$$P_c(0) - \frac{P_c(0)-P_y(0)}{l}x + \sum_{i=1}^{n} \varphi_i\,(0) \sin\left(\frac{i\pi x}{l}\right) = \\ \frac{2a\beta Q_0}{(1+\eta)g}\left(\exp\left(-\frac{(1+\eta)}{\beta}gx\right) - 1\right) + P_c(0)\exp\left(-\frac{(1+\eta)}{\beta}gx\right)$$

Multiplying both sides of the above expression bysin$\left(\frac{i\pi x}{l}\right)$and integrating it from 0 to l and taking into account that for practical values of the parameters of the system, $\left(\frac{(1+\eta)}{\beta}gx\right) << 1$ and therefore $\exp\left(-\frac{(1+\eta)}{\beta}gx\right)$ can be represented

$$\exp\left(-\frac{(1+\eta)}{\beta}gx\right) = 1 - \left(\frac{(1+\eta)}{\beta}gx\right)$$

get

$$\varphi_i(0) = -\frac{2P_c(0)}{\pi} - \frac{2P_y(0)}{\pi} + \frac{4a\beta Q_0}{\beta}\left(\frac{\pi\, l}{\left(\frac{(1+\eta)}{\beta}g\right)^2 l^2 + \pi^2}\right) - \frac{8a\beta Q_0}{(1+\eta)g} + \\ + \frac{P_c(0)}{\beta}\frac{2\pi(1+\eta)gl}{\left(\frac{(1+\eta)}{\beta}g\right)^2 l^2 + \pi^2}$$

Differentiating expressions (29) with respect to time t and substituting them into condition (26), we obtain

$$\dot{\varphi}_i(0) = 0$$

Substituting expression (29) and (9) into the first equation of expression (21), we obtain

$$\frac{P_c(t)}{l} - \frac{P_y(t)}{l} - \sum_{i=1}^{n} \frac{\pi\, i\varphi_i(t)}{l}\cos\left(\frac{i\pi x}{l}\right) = \frac{\partial Q}{\partial t} + 2aQ + \frac{1+\eta}{\beta}gP_c(t) - \\ -\frac{1+\eta}{\beta}g\frac{P_c(t)-P_y(t)}{l}x + \frac{1+\eta}{\beta}g\sum_{i=1}^{n}\varphi_i\,(t)\sin\left(\frac{i\pi x}{l}\right) \tag{32}$$

Applying Laplace transforms, from equation (32), we obtain [24,25]

$$\bar{Q} = \frac{\bar{P}_c}{l(s+2a)} - \frac{\bar{P}_y}{l(s+2a)} - \sum_{i=1}^{n}\frac{\pi\, i}{l(s+2a)}\cos\left(\frac{i\pi x}{l}\right)\bar{\varphi}_i + \frac{Q_0}{(s+2a)} - \frac{(1+\eta)g}{\beta}\frac{\bar{P}_c}{(s+2a)} + \\ + \frac{(1+\eta)gx}{\beta\, l}\frac{\bar{P}_c}{(s+2a)} - \frac{(1+\eta)gx}{\beta\, l}\frac{\bar{P}_y}{(s+2a)} - \frac{1+\eta}{\beta}g\sum_{i=1}^{n}\frac{\bar{\varphi}_i}{(s+2a)}\sin\left(\frac{i\pi x}{l}\right) \tag{33}$$

Continuity condition

$$\bar{G}_{cm} = \mathrm{f}\,\bar{\mathrm{Q}}\big|_{x=0} \tag{34}$$

taking into account expressions (16), (19) and (34), we obtain the following equation

$$\frac{G_{cm0}}{s+\alpha} + \frac{A_1 P_c(0)}{s+\alpha} - \frac{fQ_0}{(s+2a)} =$$
$$\frac{f\bar{P}_c}{l(s+2a)} - \frac{f\bar{P}_y}{l(s+2a)} - \sum_{i=1}^{n} \frac{f\pi\, i}{l(s+2a)}\bar{\varphi}_i - \frac{(1+\eta)g}{\beta}\frac{f\,\bar{P}_c}{(s+2a)} + \frac{A_1 s\bar{P}_c}{(s+2a)} \tag{35}$$

where $A_1 = \frac{k[P_k+P_c(0)]}{D\,\mu_{cm}\,\beta}\pi\, h$
From equation (35) with $i = 1$and we obtain

$$\bar{P}_c = P1_c + \left[\frac{f\,\bar{P}_y}{l(s+2a)} - \frac{2s^2 f\,\bar{P}_y}{l(s+2a)(s-\xi_1)(s-\xi_2)} - \right.$$
$$- \frac{4a\,s\,f\,\bar{P}_y}{l(s+2a)(s-\xi_1)(s-\xi_2)} -$$
$$\left. - \frac{4(1+\eta)c^2 g}{l\,\beta}\,\frac{f\,\bar{P}_y}{(s+2a)(s-\xi_1)(s-\xi_2)}\right]\frac{(s+\alpha)(s+2a)(s-\xi_1)(s-\xi_2)}{A_1(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)} \tag{36}$$

where$\beta_1,\ \beta_2,\ \beta_3$ and β_4 are the roots of the equation

$$A_1\, l^2\beta\, s(s+2a)(s-\xi_1)(s-\xi_2) + l\beta\, f(s+\alpha)(s-\xi_1)(s-\xi_2) -$$
$$-l^2 g\, f\,(1+\eta)(s+\alpha)(s-\xi_1)(s-\xi_2) + 2f\,l\,\beta\, s^2\,(s+\alpha) +$$
$$+12a\beta\, f\, l\, s(s+\alpha) + 4fgc^2(1+\eta)(s+\alpha) = 0\,,$$

$$\bar{\Phi}_1 = \frac{\varphi_{10}}{(s-\xi_1)(s-\xi_2)} + \frac{2a\,\dot{\varphi}_{10}}{(s-\xi_1)(s-\xi_2)} + \frac{2sP_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \frac{2sP_y(0)}{\pi(s-\xi_1)(s-\xi_2)} -$$
$$- \frac{12aP_c(0)}{\pi(s-\xi_1)(s-\xi_2)} + \frac{4aP_y(0)}{\pi(s-\xi_1)(s-\xi_2)} \tag{37}$$

$$P1_c = \left[\frac{f\pi\bar{\Phi}_1}{l(s+2a)} + \frac{G_{cm0}}{s+\alpha} + \frac{A_1 P_c(0)}{s+\alpha} - \frac{fQ_0}{(s+2a)}\right]\frac{(s+\alpha)(s+2a)(s-\xi_1)(s-\xi_2)}{A_1(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)}$$

Passage of the gas-liquid mixture through the choke.
The gas-liquid mixture, passing through the choke, enters the transport line. When the mixture passes through the nozzle, its pressure decreases significantly.
In the first approximation, the relationship between the flow rate of the mixture Q_{sht} and the pressure drop between the inlet and outlet of the choke is assumed to be linear [8]:

$$Q_{sht} = \alpha_0(\mathrm{P_y}(t) - P_{sht}(t)). \tag{38}$$

where α_0 is the productivity coefficient
After the Laplace transform from expression (38), we obtain

$$\bar{Q}_{sht} = \alpha_0(\bar{\mathrm{P}}_{\mathrm{y}} - \bar{P}_{sht}) \tag{39}$$

From the continuity condition, we have

$$fQ|_{x=l} = \rho_{cm} Q_{sht}. \tag{40}$$

Substituting expressions (33) and (39) into formula (40) taking into account only one term of the series, in the first approximation we obtain the following equation from which, after the Laplace transform, $P_{sht}(t)$ is determined

$$\begin{array}{l}\bar{P}_{sht} = \bar{P}_y - \frac{f\,\bar{P}_c}{\rho_{cm} l \alpha_0 (s+2a)} + \frac{f\,\bar{P}_y}{\rho_{cm} l \alpha_0 (s+2a)} - \frac{f\,\pi\,\bar{\varphi}_1}{\rho_{cm} l \alpha_0 (s+2a)} - \\ - \frac{f\,Q_0}{\rho_{cm} \alpha_0 (s+2a)} + \frac{f\,g(1+\eta)\bar{P}_y}{\rho_{cm} \alpha_0 \beta (s+2a)}\end{array} \tag{41}$$

where $P_{sht}$is the pressure at the outlet of the choke.

The movement of the gas-liquid mixture in the main pipeline.

Consider the movement of a gas-liquid mixture in a main pipeline. We place the origin of the coordinate axis x_1 at the inlet of the pipeline and direct it in the direction of the flow of the gas-liquid mixture. Suppose that at the moment $t = 0$at a distance l_2 from the wellhead, a new source with a flow rate G is connected to the main pipe. Then for the equation of motion of the gas-liquid mixture in the pipeline will have the form

$$\frac{\partial^2 P}{\partial t^2} = c^2 \frac{\partial^2 P}{\partial x_1^2} - 2a_1 \frac{\partial P}{\partial t} - \frac{2a_1 c^2 G}{f_1} \delta(x_1 - l_2). \tag{42}$$

The initial and boundary conditions

$$\left.\frac{\partial P}{\partial t}\right|_{t=0} = -c^2 \frac{G}{f_1} \delta\left(x_1 - l_2\right), 0 \leq x \leq l, \tag{43}$$

$$P(x_1, 0)|_{t=0} = P_{sht}(0) - 2a_1 Q_{cm}(0) x_1, 0 \leq x \leq l, \tag{44}$$

$$P|_{x_1=0} = P_{sht}(t), t > 0, \tag{45}$$

$$P|_{x_1=l_1} = P_2, t > 0, \tag{46}$$

where $\delta(x)$ is the Dirac function.

The solution of equation (42), taking into account the boundary conditions (45) and (46), will be sought in the form:

$$P = P_{sht}(t) - \frac{P_{sht}(t) - P_2}{l_1} x_1 + \sum_{i=1}^{n} \varphi_{2i}\left(t\right) \left(\sin \frac{i\pi x_1}{l_1}\right), \tag{47}$$

where $\varphi_{2i}\left(t\right)$ is an unknown time-dependent function t, l_1 is the length of the pipeline.

Substituting expression (47) into formula (42), taking into account the initial conditions (43), (44) similarly to (30) in the case of $P_2 = const$, we obtain a differential equation for φ_{2i}. Having solved this equation, and substituting the obtained result into the first equation of system (21) and after the Laplace transform, we obtain

$$\begin{array}{l}\left.\bar{Q}_{cm}\right|_{x_1=0} = \frac{Q_{cm}(0)}{s+2a_1} - \sum_{i=1}^{n} \left(\frac{i\pi}{l_1}\right) \frac{\bar{\varphi}_{2i}}{s+2a_1} + \frac{\bar{P}_{sht}}{l_1(s+2a_1)} \\ - \frac{P_2}{2l_1 s(s+2a_1)}\end{array}, \tag{48}$$

where $Q = \rho_{cm} u_1, u_1$, averaged over the cross-section of the flow velocity of the mixture of the transport pipeline, P_2 is the pressure at the end of the pipeline, and $\bar{\varphi}_{2i}$.

$$\begin{aligned}\bar{\varphi}_2 = \tfrac{(s+2a_1)\varphi_2(0)}{(s-\xi_3)(s-\xi_4)} - \tfrac{\dot{\varphi}_2(0)}{(s-\xi_3)(s-\xi_4)} + \tfrac{2(s+2a_1)P_{HB}(0)}{\pi(s-\xi_3)(s-\xi_4)} + \tfrac{2\dot{P}_{sht}(0)}{\pi(s-\xi_3)(s-\xi_4)} - \\ - \tfrac{2s(s+2a_1)\bar{P}_{sht}}{\pi(s-\xi_3)(s-\xi_4)} - \tfrac{4a_1c^2G}{l_1 f_1 s(s-\xi_3)(s-\xi_4)} \sin\left(\tfrac{\pi l_2}{l_1}\right)\end{aligned} \tag{49}$$

where ξ_3 and ξ_4 are the roots of the equation $s^2 + 2a_1 s + \frac{c^2\pi^2 i^2}{l_1^2} = 0$

Continuity condition

$$f_1 Q_{cm}|_{x_1=0} = \rho_{cm} Q_{sht} \tag{50}$$

Substituting expressions (39) and (48) into formula (50) taking into account only one term of the series, in the first approximation we obtain the following equation

$$\frac{f_1 Q_{cm}(0)}{s+2a_1} - \frac{\pi}{l_1}\frac{f_1\bar{\varphi}_2}{s+2a_1} + \frac{\bar{P}_{sht} f_1}{l_1(s+2a_1)} - \frac{f_1 P_2}{2l_1 s(s+2a_1)} = \rho_{cm}\alpha_0(\bar{\mathrm{P}}_{\mathrm{y}} - \bar{P}_{sht}) \tag{51}$$

From formula (51), taking into account expressions (36), (41), we obtain

$$\begin{aligned}\bar{P}_{\mathrm{y}} = \tfrac{A_4\bar{P}1_c l\beta A_1(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\alpha_7)(s-\alpha_8)}{A_6(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \\ - \tfrac{f\pi\,\bar{\Phi}_1 l^2\beta^2 A_1(s-\beta_1)(s-\beta_2)(s-\xi_1)(s-\xi_2)(s-\xi_3)(s-\xi_4)}{A_6(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \\ \tfrac{f\,Q_{cm}(0) l^3\beta^2 A_1(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\xi_1)(s-\xi_2)}{A_6(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \\ + \tfrac{f_1 Q_1(0) l^3 l_1^2\beta^3\rho_c\alpha_0 A_1(s+2a)(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\xi_1)(s-\xi_2)(s-\xi_3)(s-\xi_4)}{A_2A_6(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \\ - \tfrac{l^3 l_1\beta^3\pi\rho_c\alpha_0 A_1 f_1\bar{\Phi}_2(s+2a)(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\xi_1)(s-\xi_2)(s-\xi_3)(s-\xi_4)}{A_2A_6(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)} \\ - \tfrac{f_1 P_2}{2}\tfrac{l^3 l_1\beta^3\rho_c\alpha_0 A_1(s+2a)(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\xi_1)(s-\xi_2)(s-\xi_3)(s-\xi_4)}{A_2A_6 s(s-\alpha_1)(s-\alpha_2)(s-\alpha_3)(s-j_1)(s-j_2)(s-j_3)(s-j_4)(s-j_5)(s-j_6)(s-j_7)}\end{aligned} \tag{52}$$

where, $A_2 = l_1^2\beta\rho_c\alpha_0$,

$$\begin{aligned}\bar{\Phi}_2 = \tfrac{(s+2a_1)\varphi_2(0)}{(s-\xi_3)(s-\xi_4)} - \tfrac{\dot{\varphi}_2(0)}{(s-\xi_3)(s-\xi_4)} + \tfrac{2(s+2a_1)P_{sht}(0)}{\pi(s-\xi_3)(s-\xi_4)} + \tfrac{2\dot{P}_{sht}(0)}{\pi(s-\xi_3)(s-\xi_4)} - \\ - \tfrac{4a_1c^2G}{l_1 f_1 s(s-\xi_3)(s-\xi_4)} \sin\left(\tfrac{\pi l_2}{l_1}\right)\end{aligned} \tag{53}$$

$\alpha_1, \alpha_2, \alpha_3$ roots of the equation

$$l_1\rho_{cm}\alpha_0(s+2a_1)(s-\xi_3)(s-\xi_4) + f_1(s-\xi_3)(s-\xi_4) + 2f_1 s(s+2a_1) = 0$$

$j_1, j_2, j_3, j_4, j_5, j_6, j_7$ - roots of the equation

$$\begin{aligned}&\alpha_0^2\rho_{cm}^2\, A_1 l^3\beta^2(s+2a)(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4)(s-\xi_1)(s-\xi_2) \\ &-A_3\, l\, A_1\beta(s-\alpha_4)(s-\alpha_5)(s-\alpha_6)(s-\beta_1)(s-\beta_2)(s-\beta_3)(s-\beta_4) - \\ &-A_4\ A_5\beta(s-\alpha_7)(s-\alpha_8)(s-\alpha_9)(s-\alpha_{10})(s+\alpha) = 0\end{aligned}$$

$A_3 = l^2\beta\rho_c\alpha_0$, $\alpha_4, \alpha_5, \alpha_6$ roots of the equation

$$l^2\beta\rho_{cm}\alpha_0(s+2a)(s-\xi_1)(s-\xi_2) + f\beta\, l(s-\xi_1)(s-\xi_2) + (1+\eta)fgl^2(s-\xi_1)(s-\xi_2)+$$
$$+4l\, f\, \beta\, a\, s + 4f\, c^2 g(1+\eta) = 0$$

$A_4 = lf\beta$, α_7, α_8 roots of the equation

$$2f\beta\, ls^2 - f\beta\, l(s-\xi_1)(s-\xi_2) - 12l\, f\, \beta\, a\, s - 4f\, c^2 g(1+\eta) = 0 \qquad (54)$$

$A_5 = f\beta$, α_9, α_{10} roots of the equation

$$2f\beta\, s^2 - f\beta(s-\xi_1)(s-\xi_2) + 4\, f\, \beta\, a\, s + 4f\, c^2 g(1+\eta) = 0$$

Applying the Laplace transform and taking into account the convolution and inversion theorems [24,25], from expressions (16), (36) and (52), taking into account the numerical values of the parameters of the system

$$r_c = 0.075\ m;\quad \rho_{atm} = 0.668 : q/m^3;\quad _{atm} = 10^5\, P_a;\quad \pi = 3.14;$$
$$h = 5m;\ P_c(0) = 2.8\cdot 10^7 P0;\ P_k = 3\cdot 10^7 0;$$
$$P_y(T) = 8\cdot 10^6 P_0;\ P_0 = 2.3\cdot 10^7 P_0;\ P_y(0) = 2.3\cdot 10^7 P0;$$
$$\mu = 9\cdot 10^{-5}\, P0\cdot s\,; a = 10^{-3}s^{-1};\ m = 0.2;\ c = 300m/s;\ \ T = 180\, day;$$

we obtain

$$\begin{aligned}
&P_y = 220.741829\exp(-12.7205207\,t) - 1.77941125\cdot 10^7\exp(-0.9977667\,t)-\\
&-3.870285957\cdot 10^7\exp(-1.0\,t) + 1.801377764\cdot 10^7\exp(-0.00222335325\,t)+\\
&+7.525148748\cdot 10^5\exp(-1.06284852\,t) + 2.708110455\cdot 10^5\exp(-12.7205207\,t)+\\
&+1.508324373\cdot 10^7\exp(-0.198066191\,t) - 61.19851847\exp(-1.062851662\cdot 10^{-7}t)+\\
&+1.824140489\cdot 10^7\exp(-0.1000058782\,t)\cos(0.9366764298\,t)-\\
&-1.683680169\cdot 10^7\exp(-0.1000058782\,t)\sin(0.9366764298\,t)-\\
&-1.188600565\cdot 10^6\exp(-0.263433087\,t)\cos(0.487258856\,t)+\\
&+9.525814957\cdot 10^5\exp(-0.263433087\,t)\sin(0.487258856\,t) + 2.23287558110\cdot 10^7
\end{aligned} \qquad (55)$$

$$\begin{aligned}
&P_c = -987.698349\,t\exp(-12.7205207\,t) - 3.0006076191\cdot 10^{-7}t\exp(-1.062851662\cdot 10^{-7}t)\\
&9.2849183341\cdot 10^6\exp(-0.1000058782\,t)\cos(0.9366764298\,t)+\\
&+1.990556025\cdot 10^7\exp(-0.1000058782\,t)\sin(0.9366764298\,t)+\\
&+3.683774082\cdot 10^{11}\exp(-0.263433087\,t)\cos(0.487258856\,t)-\\
&-4.398588993\cdot 10^{11}\exp(-0.263433087\,t)\sin(0.487258856\,t) + 2.265233896\cdot 10^7-\\
&-8.8447869491\cdot 10^{11}\ \exp(-12.7205207\,t) - 1.172419217\cdot 10^{10}\exp(-1.062851662\cdot 10^{-7}\,t)-\\
&-5.568879876\cdot 10^5\exp(-0.9977667\,t) + 1.172502122\cdot 10^{10}\exp(-1.062848515\cdot 10^{-7}t)-\\
&-1.132111363\cdot 10^6\exp(-1.0\,t) + 1.752236851\cdot 10^7\exp(-0.00222335325\,t)+\\
&+8.8447486761\cdot 10^{11}\exp(-12.72052207\,t) + 1.8843259051\cdot 10^7\exp(-0.198066191\,t)
\end{aligned} \qquad (56)$$

Numerical calculations are given for the given values of the system parameters.

The calculation results are shown in Fig. 1-Fig. 3.

As can be seen from Fig. 1, the pressure at the bottom of the well at the beginning increases instantly and this drop is linear.

It can be seen from Fig. 2 that after the well start-up in the initial period, the wellhead pressure also increases instantly.

Figure 3 shows that over time, well productivity decreases and is linear. Figure 3 shows the change in well productivity at different values of the mass fraction of oil η in the mixture. As can be seen from Fig. 3, with an increase in the mass fraction of oil η in the mixture, its sharp drop occurs. For example, when the mass fraction of oil in the mixture is $\eta = 0.1$, the well productivity is equal to $1 : q/s$, and at the time when $\eta = 0.03$it grows more than 12 times, the well productivity is equal to 1 kg/s.

2. Conclusion

Based on the research carried out taking into account the dynamic
the relationship of the reservoir-well system and changes in the density of the mixture depending on the pressure, analytical formulas were obtained, with the help of which the pressure at the bottom of the well, the volume of the produced gas-liquid mixture per unit time, depending on the parameters of the reservoir-pipeline system and the mass fraction of oil at unsteady flow of gas-liquid mixture.

A model of the process of unsteady movement of a gas-liquid mixture in the reservoir-pipeline system is built when a new source is connected and fluid is withdrawn from an existing transport line. Analytical expressions have been obtained that make it possible to determine the productivity of the well, as well as the pressure at its mouth and bottom at any change in pressure at the outlet of the pipeline and connecting a new source to the existing line.

3. Denotation

P -pressure at any point in the formation; $P_c(t)$ - pressure at the bottom of the well; P_k - pressure on the formation contour; -ρ_{cm} the density of the mixture of oil and gas; r- coordinate; h- formation thickness; -m coefficient of porosity of the formation, ρ_o- oil density; ρ_q- gas density; P_{atm}- Atmosphere pressure;

ρ_{atm}- gas density at atmospheric pressure; η- mass fraction of oil in gas,

μ_{cm} - mixture viscosity, k- formation permeability coefficient,

$P_c(0)$, $P_c(T)$- pressure at the bottom hole at the beginning and end of operation,

c - speed of sound propagation in gas, t- time, x- coordinate,

a - drag coefficient, $P_C(t)$ - wellhead pressure, f-pipe flow area, $\varphi_i(t)$- unknown time t-dependent function, l- pipe running depth.

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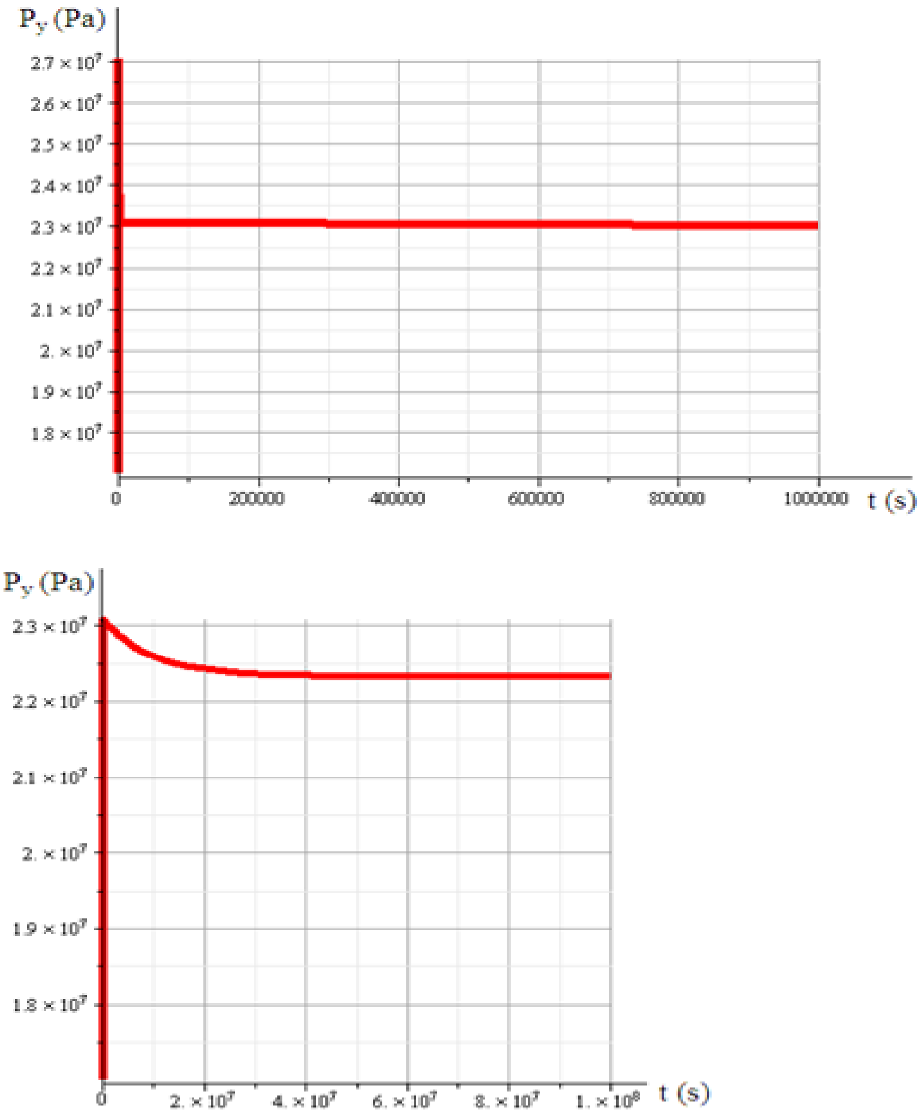


Fig. 1 - Graph of pressure dynamics at the wellhead

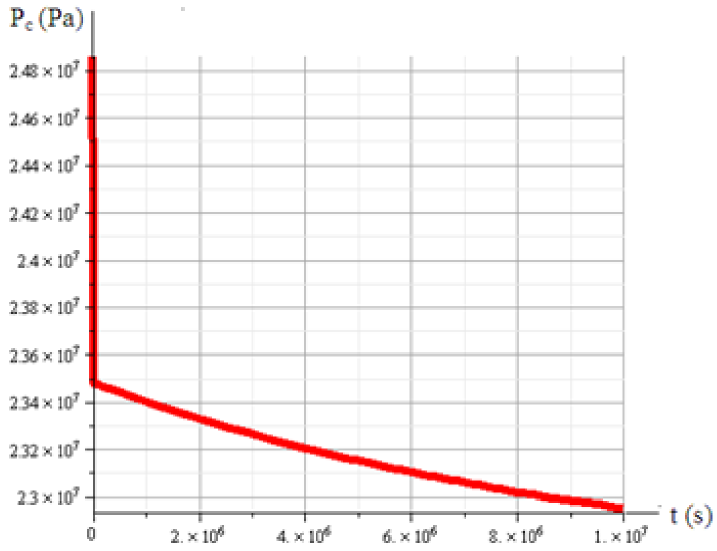
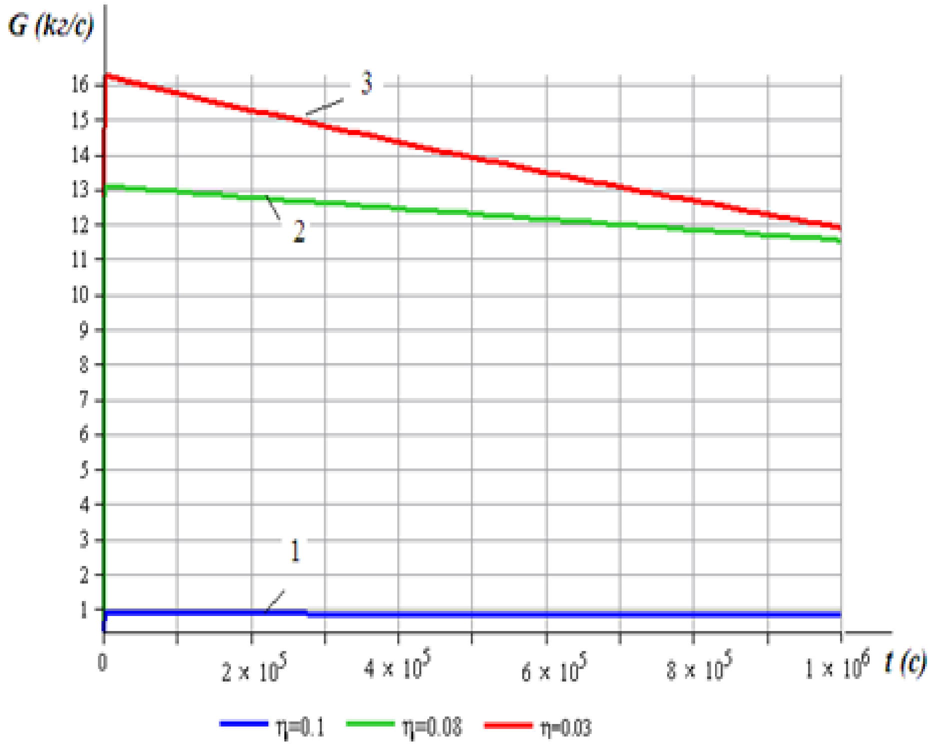


Fig. 2 - Graph of pressure dynamics at the bottom of the well



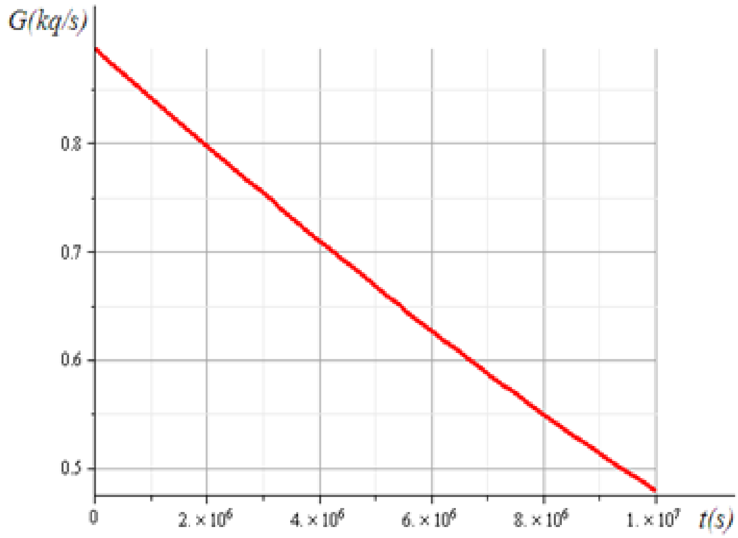


Fig. 3 Dynamics of well productivity 1-$\eta = 0.1$, 2-$\eta = 0.08$, 3-$\eta = 0.03$

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