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Mixed problem for systems of semilinear hyperbolic equations with anisotropic elliptic part nonlinear dissipations

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Abstract. In this paper we investigate the mixed problem for some class of quasi linear hyperbolic equations with nonlinear dissipation and with anisotropic elliptic part. The theorems of local solution and global solution are proved.

Key Words and Phrases: global solution, hyperbolic system, existence, anisotropic elliptic parts.

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1. Introduction

The solution of a series of technical problems is brought to non-stationary equations with derivatives of a different order by space variables [1, 2]. For these equations, the problem with initial conditions in time reduces to abstract hyperbolic equations in some function spaces. Those terms of these equations in which only derivatives with respect to space variables participate are called the anisotropic elliptic part.

In this paper, we study a mixed problem for systems of hyperbolic equations with an anisotropic elliptic part in a certain cylinder whose base is a certain three-dimensional cube. The existence and uniqueness of local and global solutions of this mixed problem with Dirichlet boundary conditions are proved.

2. Statement of the problem and main results

Let us introduce the following notation: $x = (x_1, x_2, x_3) \in \Pi_3$,

$$x_1(a) = (a, x_2, x_3), x_2(a) = (x_1, a, x_3), \quad x_3(a) = (x_1, x_2, a).$$

Let us also introduce the notation:

$$\langle u, v\rangle = \int_{\Pi_3} u\,(x)\, v\,(x)\, dx, u, v \in L_2\,(\Pi_3)\,, \|u\| = \sqrt{\langle u, u\rangle}.$$

Let us consider the mixed problem for systems of semilinear equations

$$\left.\begin{array}{l} u_{1tt} + \sum_{k=1}^{3} (-1)^{L_{1k}} D_{x_k}^{2\Lambda_{1k}} u_1 + |u_{1t}|^{r_1 - 1} u_{1t} = g_1 (u_1, u_2) \\ u_{2tt} + \sum_{k=1}^{3} (-1)^{\mathrm{L}_{2k}} D_{x_k}^{2\Lambda_{2k}} u_2 + |u_{2t}|^{r_2 - 1} u_{2t} = g_2 (u_1, u_2) \end{array}\right\} \tag{1}$$

with boundary conditions

$$D_{x_k}^{\beta_k} u_1 (t, x_k (0)) = D_{x_k}^{\beta_k} u_1 (t, x_k (1)) = 0, \quad \beta_k = 0, 1, ..., \Lambda_{ik} - 1, i = 1, 2, \quad k = 1, 2, 3, \tag{2}$$

and initial conditions

$$u_i (0, x) = \varphi_i (x), u_{it} (0, x) = \psi_i (x), \quad x \in \Pi_3, i = 1, 2. \tag{3}$$

where $\Lambda_{ik} \in N,\ i = 1, 2,\ \ k = 1, 2, 3,\ \ g_1$ and g_2 are the following non-linear functions

$$g_1 (u_1, u_2) = a_1 |u_1 + u_2|^{p_1 + p_2} (u_1 + u_2) + b_1 |u_1|^{p_1 - 1} |u_2|^{p_2 + 1} u_1,$$

$$g_2 (u_1, u_2) = a_2 |u_1 + u_2|^{p_1 + p_2} (u_1 + u_2) + b_2 |u_1|^{p_1 + 1} |u_2|^{p_2 - 1} u_2,$$

$a_1, a_2, b_1, b_2, p_1, p_2$ are real constants and

$$p_1 \geq 0, \ \ p_2 \geq 0. \tag{4}$$

We introduce the notation: $\left|\overrightarrow{\Lambda_i}^{-1}\right| = \sum_{k=1}^{3} \frac{1}{\Lambda_{ik}}$, where $\overrightarrow{\Lambda_i} = (\Lambda_{i1}, \Lambda_{i2}, \Lambda_{i3})$. Let us denote the anisotropic Sobolev space by $W_2^{\overrightarrow{\Lambda_i}}$, i.e.

$$W_2^{\overrightarrow{\Lambda_i}} = W_2^{\overrightarrow{\Lambda_i}}(\Pi_3) = \left\{ v : \ v, D_{x_k}^{\Lambda_{ik}} v \in L_2(\Pi_3) \right\},$$

$$\|v\|_{W_2^{\overrightarrow{\Lambda_i}}} = \left[\|v\|_{L_2(\Pi_3)}^2 + \sum_{k=1}^{n} \left\| D_{x_k}^{\Lambda_{ik}} v \right\|_{L_2(\Pi_3)}^2 \right]^{1/2}.$$

Denote by $\hat{W}_2^{\overrightarrow{\Lambda_i}}$ the next subspace of $W_2^{\overrightarrow{\Lambda_i}}$:

$$\hat{W}_2^{\overrightarrow{\Lambda_i}} = \left\{ u : \ u \in W_2^{\overrightarrow{\Lambda_i}}, D_{x_k}^{\beta_k} u(t, x_k(0)) = D_{x_k}^{\beta_k} u(t, x_k(1)) = 0, \beta_k = 0, 1, ..., \Lambda_{ik} - 1 \right\}.$$

Let X be some Banach space and denote by $C ([0, T]; X)$ the set of continuous functions acting from $[0, T]$ to X: $\|u (t)\|_{C([0,T];X)} = \max\limits_{0 \leq t \leq T} \|u (t)\|_X$.

Denote by $C^k ([0, T]; X)$ the set of continuously differentiable functions of order k acting from $[0, T]$ to X: $\|u (t)\|_{C^k([0,T];X)} = \sum_{i=0}^{k} \left\|u^{(i)} (t)\right\|_{C([0,T];X)}$.

Denote by $C_w ([0, T]; X)$ the set of weakly continuous functions acting from $[0, T]$ to X.

Let us define the following spaces of functions

$$H_T^1 = C \left([0, T]; \hat{W}_2^{\overline{\Lambda}_1} \times \hat{W}_2^{\overline{\Lambda}_2} \right) \cap C^1 ([0, T]; L_2 (\Pi_3) \times L_2 (\Pi_3)),$$

$$H^1_{T,\infty}=\left\{u:\quad u\in L_\infty\left(0,T;\hat W_2^{\overline{\Lambda}_1}\times \hat W_2^{\overline{\Lambda}_2}\right),u_t\in L_\infty\left(0,T;L_2\left(\Pi_3\right)\times L_2\left(\Pi_3\right)\right)\right\},$$

$$H^2_{T,w}=\{u:\quad u\in C_w\left(\left[0,T\right];\hat W_2^{2\overline{\Lambda}_1}\times \hat W_2^{2\overline{\Lambda}_2}\right),\quad u_t\in C_w\left(\left[0,T\right];\hat W_2^{\overline{\Lambda}_1}\times \hat W_2^{\overline{\Lambda}_2}\right)\quad,$$

$$u_{tt}\in C_w\left(\left[0,T\right];L_2\left(\Pi_3\right)\times L_2\left(\Pi_3\right)\right)\},$$

$$H^2_{T,\infty}=\left\{u:\quad u\in L_\infty\left(0,T;\hat W_2^{2\overline{\Lambda}_1}\times \hat W_2^{2\overline{\Lambda}_2}\right),u_t\in L_\infty\left(0,T;\hat W_2^{\overline{\Lambda}_1}\times \hat W_2^{\overline{\Lambda}_2}\right)\right.$$

$$\left.u_{tT}\in L_\infty\left(0,T;L_2\left(\Pi_3\right)\times L_2\left(\Pi_3\right)\right)\right\}.$$

It is clear from the expression of the functions $g_i(u_1,u_2)$, that

$$\left|g_i\left(u_1,u_2\right)\right|\le c\left[\ \left|u_1\right|^{p_1+p_2+1}+\left|u_2\right|^{p_1+p_2+1}\right],\ i=1,2,\ c>0. \tag{5}$$

A strong solution of problem (1) - (3) is a pair of function $s\left(u_1\left(\cdot\right),u_2\left(\cdot\right)\right)\in H^2_{T,\infty}$, such that for all $(\eta_1(\cdot),\eta_2(\cdot))\in \hat W_2^{\overline{\Lambda}_1}\times \hat W_2^{\overline{\Lambda}_2}$ the following equalities hold

$$a)\ \frac{d}{dt}\left\langle u_{1t}\left(t,\cdot\right),\eta_1\left(\cdot\right)\right\rangle+\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{1k}}u_1\left(t,\cdot\right),D_{x_k}^{\Lambda_{1k}}\eta_1\left(\cdot\right)\right\rangle+$$

$$+\left\langle\left|u_{1t}\left(t,\cdot\right)\right|^{r_1-1}u_{1t}\left(t,\cdot\right),\eta_1\left(\cdot\right)\right\rangle=\left\langle g_1(u_1\left(t,\cdot\right),u_2\left(t,\cdot\right),\eta_1\left(\cdot\right)\right\rangle, \tag{6}$$

$$\frac{d}{dt}\left\langle u_{2t}\left(t,\cdot\right),\eta_2\left(\cdot\right)\right\rangle+\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{2k}}u_2\left(t,\cdot\right),D_{x_k}^{\Lambda_{1k}}\eta_2\left(\cdot\right)\right\rangle+$$

$$+\left\langle\left|u_{2t}\left(t,\cdot\right)\right|^{r_1-1}u_{2t}\left(t,\cdot\right),\eta_2\left(\cdot\right)\right\rangle=\left\langle g_2(u_1\left(t,\cdot\right),u_2\left(t,\cdot\right),\eta_2\left(\cdot\right)\right\rangle,$$

$$\text{almost all } t\in(0,T), \tag{7}$$

$$b)\ \lim_{t\to+0}\left\|\sum_{k=1}^{3}D_{x_k}^{\Lambda_{ik}}\left[u_i\left(t,\cdot\right)-\varphi_i\left(\cdot\right)\right]\right\|_{L_2(\Pi_3)}=0,i=1,2, \tag{8}$$

$$c)\ \lim_{t\to+0}\int_{\Pi_3}\sum_{k=1}^{3}D_{x_k}^{\Lambda_{ik}}\left[u_{it}\left(t,x\right)-\psi_i\left(x\right)\right]D_{x_k}^{\Lambda_{ik}}\eta_i\left(x\right)dx=0,\quad i=1,2. \tag{9}$$

By a weak solution to problem (1) - (3) we mean the functions $\left(u_1\left(\cdot\right),u_2\left(\cdot\right)\right)\in H^1_{T,\infty}$ such that for all $\left(\eta_1\left(\cdot\right),\eta_2\left(\cdot\right)\right)\in H^1_{T,\infty}$, $\eta_i\left(x,T\right)=0,i=1,2$ the following equalities hold

$$a)\int_0^T\left[\left\langle u_{it}\left(t,\cdot\right),\eta_{it}\left(t,\cdot\right)\right\rangle+\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{1k}}u_i\left(t,\cdot\right),D_{x_k}^{\Lambda_{1k}}\eta_i\left(\cdot\right)\right\rangle\right]dt+$$

$$+\int_0^T\left\langle\left|u_{it}\left(t,\cdot\right)\right|^{r_i-1}u_{it}\left(t,\cdot\right),\eta_i\left(t,\cdot\right)\right\rangle dt=$$

$$= \int_0^T \langle g_i(u_1(t,\cdot), u_2(t,\cdot)), \eta_i(t,\cdot)\rangle\, dt + \langle \psi_i(\cdot), \eta_i(0,\cdot)\rangle, i = 1,2, \quad (10)$$

$$b) \lim_{t\to 0} \langle u_i(\cdot,t) - \phi_i(\cdot), \eta_1(t,\cdot)\rangle_{\hat{W}_2^{\overline{\Lambda}_i}} = 0, \quad i = 1,2. \quad (11)$$

It is known that under the condition

$$min\left\{\left|\overline{\Lambda}_1^{-1}\right|, \left|\overline{\Lambda}_2^{-1}\right|\right\} > 2, \quad (12)$$

the embedding

$$\hat{W}_2^{\overline{\Lambda}_i} \subset C(\overline{\Pi}_3), \quad i = 1,2, \quad (\text{see } [3]) \quad (13)$$

is valid. The following theorems on the existence of a local solution of the problem (1) - (3) are true.

Theorem 1. *Suppose that the conditions* (4), (5) *and* (12) *are satisfied. Then for any initial data* $(\phi_1, \phi_2) \in \hat{W}_2^{2\overline{\Lambda}_1} \times \hat{W}_2^{2\overline{\Lambda}_2}$, $(\psi_1, \psi_2) \in \hat{W}_2^{\overline{\Lambda}_1} \times \hat{W}_2^{\overline{\Lambda}_2}$ *there exists* $T' > 0$ *such that the problem* (1) - (4) *has a unique solution* $(u_1, u_2) \in H^2_{T',w}$.

In addition, if $T_{\max} = \max T'$ is the length of the maximum interval of the existence of this solution, then one of the following statements is true:

$$\lim_{t\to T_{max}} \sum_{i=1}^{2} \left[\|u_{it}(t,\cdot)\|^2 + \|u_1(t,\cdot)\|^2_{\hat{W}_2^{\overline{\Lambda}_i}}\right] = +\infty; \quad (14)$$

or

$$T_{\max} = +\infty. \quad (15)$$

Theorem 2. *Suppose that the conditions* (4), (5) *and* (12) *are satisfied. Then for any initial data* $(\phi_1, \phi_2) \in \hat{W}_2^{\overline{\Lambda}_1} \times \hat{W}_2^{\overline{\Lambda}_2}$, $(\psi_1, \psi_2) \in L_2(\Pi_3) \times L_2(\Pi_3)$ *there exists* $T' > 0$ *such that the problem* (1) - (3) *has a unique solution* $(u_1, u_2) \in H^1_{T',w}$.

In addition, if $T_{\max} = \max T'$ is the length of the maximum interval of the existence of this solution, then one of the relations (14) and (15) is true.

In some cases, for any $T > 0$, the local solutions defined by Theorem 1 can be distributed over the entire $[0,T] \times \Pi_3$ region. According to Theorem 1, this is possible if the following a priori estimate is true for local solutions

$$\sum_{i=1}^{2} \left[\|u_{it}(t,x)\|^2 + \left\|\sum_{k=1}^{3} D_{x_k}^{\Lambda_{ik}} u_i(t,x)\right\|^2\right] \le c, 0 \le t \le T \quad (16)$$

We get this estimate if

$$\lambda = \frac{a_1(p_1+1)}{b_1} = \frac{a_2(p_2+1)}{b_2}, \quad (17)$$

$$a_i \le 0, b_i \le 0, \quad i = 1,2. \quad (18)$$

When these conditions are met, the following theorem on the global solvability of the problem (1) - (3) is proved.

Theorem 3. *Suppose that the conditions* (4) , (5), (17) *and* (18) *are satisfied, then for any* $T>0$, $(\phi_1,\phi_2)\in \hat{W}_2^{\overline{\Lambda}_1}\times \hat{W}_2^{\overline{\Lambda}_2}$ *and* $(\psi_1,\psi_2)\in L_2(\Pi_3)\times L_2(\Pi_3)$ *the problem* (1) - (3) *has a unique solution* $(u_1(\cdot),u_2(\cdot))\in C\left([0,T];\hat{W}_2^{\overline{\Lambda}_1}\times \hat{W}_2^{\overline{\Lambda}_2}\right)\cap C^1([0,T];L_2(\Pi_3)\times L_2(\Pi_3))$.

3. Proof of Theorem 1

We will prove the theorem using Galyorkin's method. Let $e_j(x)$, $j=1,2,...$-denote the solutions of the following problem:

$$\sum_{k=1}^{3}(-1)^{\Lambda_{ik}}D_{x_k}^{2\Lambda_{ik}}e_{ij}(x)=\lambda_{ij}e_{ji}(x),x\in\Pi_3,$$

$$D_{x_k}^{\beta_k}e_j(x_k(0))=D_{x_k}^{\beta_k}e_j(x_k(1))=0,\beta_k=0,1,...,\Lambda_{ik}-1,k=1,...,n,\ i=1,2.$$

In other words, $e_{ij}(x)$ $,x\in\Pi_3$, $j=1,2,$ $i=1,2,...$ are eigenfunctions of the operator $\overrightarrow{\mathcal{L}}=\sum_{k=1}^{3}(-1)^{\Lambda_{ik}}D_{x_k}^{2\Lambda_{ik}}$ with the Dirichlet boundary condition (see [4, 5]).

We approximate the functions $\varphi_i(x)$ and $\psi_i(x)$ and the functions $\varphi_{im}(x)$ and $\psi_{im}(x)$, $i=1,2$, $m=1,2,...$ respectively. So that,

$$\phi_{im}=\sum_{r=1}^{m}a_{irm}e_{ir}(x)\to\phi_i,\ \text{in } \hat{W}_2^{2\overline{\Lambda}_i}\text{ as } m\to\infty,i=1,2, \tag{19}$$

$$\psi_{im}=\sum_{r=1}^{m}b_{irm}e_{ir}(x)\to\psi_i,\ \text{in } \hat{W}_2^{\overline{\Lambda}_i}\text{ as } m\to\infty,i=1,2. \tag{20}$$

We are looking for approximate solutions of problem (1) - (3) as follows

$$u_{im}(t,x)=\sum_{r=1}^{m}C_{irm}(t)e_{ir}(x),\quad i=1,2,$$

so that the functions $C_{irm}(t)$, $i=1,2,$ $r=1,...,m$ are the solutions of the following Cauchy problem for the system of ordinary differential equations

$$\langle u_{im_{tt}}(t,x),e_{ir}(x)\rangle+\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{ik}}u_{im}(t,x),D_{x_k}^{\Lambda_{ik}}e_{ir}(x)\right\rangle+$$

$$+\int_{\Pi_3}|u_{im_t}(t,x)|^{r_1-1}u_{im_t}(t,x)e_{ir}(x)dx=\langle g_1(u_{1m}(t,x),u_{2m}(t,x)),e_{ir}(x)\rangle,$$

$$r=1,\dots,m,\quad i=1,2, \tag{21}$$

$$u_{im}(0,x)=\phi_{im}(x),u_{im_t}(0,x)=\psi_{im}(x),\quad x\in\Pi_n,\quad i=1,2. \tag{22}$$

According to Cauchy-Picard theorem [6], on the existence of a solution of the Cauchy problem for a system of ordinary differential equations, problem (21) - (22) has a solution in some half-interval $[0,t_m)$.

Multiplying both side of each equation (21) by the function $C'_{ir}(t)$, and summing up the resulting equalities, we obtain

$$\langle u_{im_{tt}}(t,x), u_{imt}(t,x)\rangle + \sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{ik}} u_{im}(t,x), D_{x_k}^{\Lambda_{ik}} u_{imt}(t,x)\right\rangle$$

$$+\int_{\Pi_3} |u_{im_t}(t,x)|^{r_1+1} dx =$$

$$= \langle g_i(u_{1m}(t,x), u_{2m}(t,x)), u_{im_t}(t,x)\rangle, i = 1,2. \tag{23}$$

It is obvious that

$$\langle u_{im_{tt}}(t,x), u_{im_t}(t,x)\rangle = \frac{1}{2}\frac{d}{dt}\|u_{im_t}(t,\cdot)\|^2, \quad i=1,2, \tag{24}$$

$$\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{ik}} u_{1m}(t,x), D_{x_k}^{\Lambda_{ik}} u_{1mt}(t,x)\right\rangle = \frac{1}{2}\frac{d}{dt}\sum_{k=0}^{3}\left\|D_{x_k}^{\Lambda_{ik}} u_{im}(t,\cdot)\right\|^2, \ i=1,2. \tag{25}$$

Summing equalities (23) and taking into account (24) and (25) , we obtain:

$$\frac{d}{dt}\sum_{i=1}^{2}\left[\|u_{im_t}(t,\cdot)\|^2 + \sum_{k=1}^{3}\left\|D_{x_k}^{\Lambda_{ik}} u_{im}(t,\cdot)\right\|^2\right] + \sum_{i=1}^{2}\int_{\Pi_n} |u_{im_t}(t,x)|^{r_i+1} dx =$$

$$= \sum_{i=1}^{2}\int_{\Pi_3} g_i(u_{1m}(t,x), u_{2m}(t,x)), u_{im_t}(t,x) dx. \tag{26}$$

Using the Hölder's and Young's inequalities, we obtain the following:

$$\left|\int_{\Pi_3} g_i(u_{1m}(t,x), u_{2m}(t,x)), u_{im_t}(t,x) dx\right| \le$$

$$\le \left(\frac{1}{(r_i+1)\varepsilon}\right)^{\frac{1}{r_i}} \int_{\Pi_3} |g_i(u_{1m}(t,x), u_{2m}(t,x))|^{\frac{r_i+1}{r_i}} dx + \varepsilon\int_{\Pi_3} |u_{im_t}(t,x)|^{r_i+1} dx.$$

Using (5) we have

$$\int_{\Pi_3} |g_i(u_{1m}(t,x), u_{2m}(t,x))|^{\frac{r_i+1}{r_i}} dx \le$$

$$\le C\left[\int_{\Pi_3} |u_1|^{(p_1+p_2+1)\frac{r_i+1}{r_i}} dx + \int_{\Pi_3} |u_2|^{(p_1+p_2+1)\frac{r_i+1}{r_i}} dx\right] \le$$

$$\le C\left[\|u_1\|_{C(\overline{\Pi}_3)}^{(p_1+p_2+1)\frac{r_i+1}{r_i}} + \|u_2\|_{C(\overline{\Pi}_3)}^{(p_1+p_2+1)\frac{r_i+1}{r_i}}\right] \le C\sum_{i=1}^{2}\|u_i\|_{\hat{W}_2^{\overline{\Lambda}_i}}^{(p_1+p_2+1)\frac{r_i+1}{r_i}}. \tag{27}$$

It follows from (26) and (27) that

$$\frac{d}{dt}\sum_{i=1}^{2}\left[\left\|u_{im_t}(t,\cdot)\right\|^2+\sum_{k=1}^{3}\left\|D_{x_k}^{\Lambda_{ik}}u_{im}(t,\cdot)\right\|^2\right]+(1-\varepsilon)\sum_{i=1}^{2}\int_{\Pi_3}\left|u_{im_t}(t,x)\right|^{r_i+1}dx\le$$

$$\le C\sum_{i=1}^{2}\left\|u_i\right\|_{\hat{W}_2^{\bar{\Lambda}_i}}^{(p_1+p_2+1)\frac{r_i+1}{r_i}}. \tag{28}$$

Hence, for

$$y=y(t)=\sum_{i=1}^{2}\left[\left\|u_{im_t}(t,\cdot)\right\|^2+\sum_{k=1}^{3}\left\|D_{x_k}^{\Lambda_{ik}}u_{im}(t,\cdot)\right\|^2\right]$$

we obtain the following inequality

$$y^{'}\le C\sum_{i=1}^{2}y^{(p_1+p_2+1)\frac{r_i+1}{r_i}}.$$

From here we obtain the following inequality

$$z'\le C_1z^p,\, z(0)=z_0=y_0+1, \tag{29}$$

where $z=z(t)=y(t)+1$, $p=(p_1+p_2+1)\,,\max\left\{\frac{r_1+1}{r_1},\frac{r_2+1}{r_2}\right\}$.
From inequality (29) we obtain that

$$y\le\frac{y_0+1}{\left[1-c_1(p-1)(y_0+1)^{p-1}t\right]^{\frac{1}{p-1}}}-1.$$

It follows that

$$y(t)\le 2(y_0+1)\,,0\le t\le T', \tag{30}$$

where $T'=\frac{1}{2c_1(p-1)(y_0+1)^{p-1}}$.
From (30) we obtain the following a priori estimate:

$$\sum_{i=1}^{2}\left[\left\|u_{im_t}(t,\cdot)\right\|^2+\sum_{k=1}^{3}\left\|D_{x_k}^{\Lambda_{ik}}u_{im}(t,\cdot)\right\|^2\right]\le$$

$$\le c_1\sum_{i=1}^{2}\left[\left\|\psi_{im}\right\|^2+\sum_{k=0}^{n}\left\|D_{x_k}^{\Lambda_{ik}}\phi_{im}\right\|^2\right],0\le t\le T^{'}. \tag{31}$$

According to (19), (20), we get

$$\sum_{i=1}^{2}\left[\left\|\psi_{im}\right\|^2+\sum_{k=0}^{3}\left\|D_{x_k}^{\Lambda_{ik}}\phi_{im}\right\|^2\right]\le c_2. \tag{32}$$

From (31) and (32) it follows that

$$\sum_{i=1}^{2}\left[\|u_{im_t}(t,\cdot)\|^2+\sum_{k=1}^{3}\left\|D_{x_k}^{\Lambda_{ik}}u_{im}(t,\cdot)\right\|^2\right]\leq c_3, \tag{33}$$

where $c_3>0$ is a constant independent of m.
It follows from (28) and (33) that

$$\sum_{i=1}^{2}\int_0^t\int_{\Pi_3}|u_{im_s}(s,x)|^{r_i+1}dxds\leq c_4, 0\leq t\leq T, \tag{34}$$

where $c_i>0,\ i=1,2,3$ are constants that do not depend on m.
Multiplying both sides of (21) by the function $C_{ik}^{''}(t)$, summing over $k=1$ to m, we get that:

$$\begin{aligned}\|u_{im_{tt}}(t,\cdot)\|^2\leq\|u_{im}(t,\cdot)\|_{\hat W_2^{2\vec\Lambda}}\cdot\|u_{imtt}(t,x)\|+\\+\left(\int_{\Pi_3}|u_{im_t}(t,x)|^{2r_1}dx\right)^{\frac{1}{2}}\|u_{imtt}(t,x)\|+\\+\left(\int_{\Pi_3}|g_i(u_{1m}(t,x),u_{2m}(t,x))|^2dx\right)^{\frac{1}{2}}\|u_{imtt}(t,x)\|\leq\\\leq\|u_{im}(t,\cdot)\|_{\hat W_2^{2\vec\Lambda}}\cdot\|u_{imtt}(t,x)\|+\\+\left(\max_{x\in\overline\Pi_3}\lceil u_{im_t}(t,x)\rceil\right)^{r_1}\|u_{imtt}(t,x)\|+\\+\max_{x\in\overline\Pi_3}\lceil g_i(u_{1m}(t,x),u_{2m}(t,x))\rceil\ \|u_{imtt}(t,x)\|\leq\\\leq\delta\|u_{imtt}(t,x)\|+c_\delta\|u_{im}(t,\cdot)\|_{\hat W_2^{2\vec\Lambda}}.\end{aligned}$$

From this relation it follows that

$$\|u_{im_{tt}}(0,\cdot)\|\leq C\|\phi_{im}\|_{\hat W_2^{2\vec\Lambda}}, i=1,2. \tag{35}$$

We differentiate both parts (21) - by t. Then we multiply each of the obtained equations by $c_{ikmtt}(t)$ and add them. Then we will get the following equality

$$\begin{aligned}\langle u_{imttt}(t,\cdot),u_{1m_{tt}}(t,\cdot)\rangle+\sum_{k=1}^{3}\left\langle D_{x_k}^{\Lambda_{ik}}u_{1mt}(t,x),D_{x_k}^{\Lambda_{ik}}u_{1mtt}(t,x)\right\rangle+\\+\int_{\Pi_3}\frac{\partial}{\partial t}\left(|u_{im}(t,x)|^{r_i-1}u_{im}(t,x)\right)u_{im_{tt}}(t,x)dx=\\=\sum_{j=1}^{2}\left\langle g_{iu_j}(u_{1m}(t,x),u_{2m}(t,x))u_{jm_t}(t,x),u_{jm_{tt}}(t,x)\right\rangle.\end{aligned} \tag{36}$$

Since $\beta(s) = |s|^{\gamma-1} s$ is a monotonically increasing function, therefore

$$\int_{\Pi_3} \frac{\partial}{\partial t} \left(|u_{im}(t,x)|^{r_i-1} u_{im}(t,x) \right) u_{im_{tt}}(t,x)\, dx \geq 0. \tag{37}$$

If we evaluate the right side of the equality (36) from above, we get that:

$$|J_j| = \left| \left\langle g_{iu_j}(u_{1m}(t,x), u_{2m}(t,x))\, u_{jm_t}(t,x), u_{jm_{tt}}(t,x) \right\rangle \right| \leq$$

$$\leq \left(\int_{\Pi_3} \left| g_{iu_j}(u_{1m}(t,x), u_{2m}(t,x)) \right|^2 |u_{jm_t}(t,x)|^2 dx \right)^{\frac{1}{2}} \left(\int_{\Pi_3} |u_{jm_{tt}}(t,x)|^2 dx \right)^{\frac{1}{2}}. \tag{38}$$

In view of the embedding theorems, using (12) , we obtain that $g_{iu_j}(u_{1m}(t,x), u_{2m}(t,x))$ $\in C\left(\overline{\Pi}_3\right)$. That is why

$$|J_j| = \sup_{x\in\overline{\Pi}_3} \left| g_{iu_j}(u_{1m}(t,x), u_{2m}(t,x)) \right| \left(\int_{\Pi_3} |u_{jm_t}(t,x)|^2 dx \right)^{\frac{1}{2}} \left(\int_{\overline{\Pi}_3} |u_{im_{tt}}(t,x)|^2 dx \right)^{\frac{1}{2}} \leq$$

$$\leq c \left(\sup_{x\in\widehat{\overline{\Pi}}_3} |u_{1m}(t,x)|^{p_1+p_2} + \sup_{x\in\widehat{\overline{\Pi}}_3} |u_{2m}(t,x)|^{p_1+p_2} \right) \|u_{im_t}(t,\cdot)\| \cdot \ \|u_{im_{tt}}(t,\cdot)\| \leq$$

$$\leq C \sum_{i=1}^{2} \|u_{im}(t,\cdot)\|_{\hat{W}_2^{\overline{\Lambda}_i}} \ \|u_{im_t}(t,\cdot)\| \, \|u_{im_{tt}}(t,\cdot)\|. \tag{39}$$

Taking into account (37) and (39) in (36), we obtain the inequality

$$\frac{\partial}{\partial t} \sum_{i=1}^{2} \left[\|u_{im_{tt}}(t,\cdot)\|^2 + \|u_{imt}(t,\cdot)\|_{\hat{W}_2^{\overline{\Lambda}_i}} \right] \leq c \sum_{i=1}^{2} \|u_{im_{tt}}(t,\cdot)\|^2. \tag{40}$$

From here we get

$$\sum_{i=1}^{2} \{ \|u_{im_{tt}}(t,\cdot)\|^2 + \|u_{imt}(t,\cdot)\|_{\hat{W}_2^{\overline{\Lambda}_i}} \} \leq c. \tag{41}$$

If we multiply both side (21) by $\lambda_j c_{jkm}(t,\cdot)$ and sum over $j = 1, ..., m$ and $k = 1,2,3$, we get the following equality

$$\left\langle u_{im_{tt}}(t,x), \sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{1m}(t,x) \right\rangle + \left\langle \sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x), \sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x) \right\rangle +$$

$$+ \int_{\Pi_3} |u_{im_t}(t,x)|^{r_i-1} u_{im_t}(t,x), \sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x) dx =$$

$$= \left\langle g_i(u_{1m}(t,x), u_{2m}(t,x)), \sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x) \right\rangle. \tag{42}$$

From here, using the Hölder inequality, we obtain that

$$\left\|\sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x)\right\|^2 \le \|u_{im_{tt}}(t,\cdot)\| \left\|\sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x)\right\| +$$

$$+\left\|\sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{im}(t,x)\right\| \left(\int_{\Pi_3} |u_{1m_t}(t,x)|^{2r_1} dx \right)^{1/2} +$$

$$+\left\|\sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{1m}(t,x)\right\| \left(\int_{\Pi_3} |g_1(u_{1m}(t,x), u_{2m}(t,x))|^2 dx \right)^{1/2}.$$

Taking into account a priori estimates (41) from here we get

$$\left\|\sum_{k=1}^{3} D_{x_k}^{2\Lambda_{ik}} u_{1m}(t,\cdot)\right\| \le c. \tag{43}$$

By virtue of (41) - (43) there is a subsequence of $\{u_{1m_k}, u_{2m_k}\}$ which we will denote by $\{u_{1m}, u_{2m}\}$, where

$$u_{im} \to u_i \text{ *-weak in } L_\infty\left(0,T;\hat{W}_2^{2\overline{\Lambda}_i}\right), i=1,2, \tag{44}$$

$$u_{im_t} \to u_{it} \text{ *-weak in } L_\infty\left(0,T;\hat{W}_2^{\overline{\Lambda}_i}\right), i=1,2. \tag{45}$$

$$u_{im_t} \to u_{it} \text{ *-weak in } L^{r_i+1}\left((0,T)\times\Pi_3\right), i=1,2, \tag{46}$$

$$u_{im_{tt}} \to u_{itt} \text{ *-weak in } L_\infty\left(0,T;L_2(\Pi_3)\right), i=1,2. \tag{47}$$

It follows from (44) and (45) that

$$u_i \in C\left([0,T];\hat{W}_2^{\overline{\Lambda}_i}\right), i=1,2. \tag{48}$$

On the other hand, it is known that if $u_1, u_2 \in C\left([0,T];\hat{W}_2^{\overline{\Lambda}_i}\right) \cap L_\infty\left(0,T;\hat{W}_2^{2\overline{\Lambda}_i}\right)$, where $(u_1(\cdot), u_2(\cdot))$ is a solution of the problem (1)-(3) then $u_1, u_2 \in C_w\left([0,T];\hat{W}_2^{2\overline{\Lambda}_i}\right)$ (see [4, 7]). Similarly, we can show that

$$u_{1t} \in C_w\left([0,T];\hat{W}_2^{\overline{\Lambda}_i}\right) \text{ and } u_{itt} \in C_w\left([0,T];L_2(\Pi_3)\right). \tag{49}$$

If in (19) we pass to the limit as $m\to\infty$, then we obtain that the functions (u_1, u_2) satisfy the systems (1).

According to (48), (49), these functions also satisfy the initial conditions (2), (3).

4. Proof of Theorem 2

We choose such functions $\phi_{ik}\in \hat{W}_2^{2\overline{\Lambda}_i}$, $\psi_{ik}\in \hat{W}_2^{\overline{\Lambda}_i}$, $i=1,2,\ k=1,2,...$ that

$$\left.\begin{array}{lll} \phi_{ik}\to\phi_i & in & \hat{W}_2^{\overline{\Lambda}_i} \\ \psi_{ik}\to\psi_i & in & L_2\left(\Pi_3\right) \end{array}\right\} \tag{50}$$

as $k\to\infty$.
Then, according to Theorem 1, there exist functions $(u_{1r},u_{2r})\in H^2_{T_r}$, $r=1,2,...$ such that

$$\left.\begin{array}{l} u_{1rtt}+\sum_{k=1}^n(-1)^{\Lambda_{1k}}D_{x_k}^{2\Lambda_{1k}}u_{1r}+|u_{1rt}|^{r_1-1}u_{1rt}=g_1\left(u_{1r},u_{2r}\right) \\ u_{2rtt}+\sum_{k=1}^n(-1)^{\Lambda_{2k}}D_{x_k}^{2\mathrm{L}_{2k}}u_{2r}+|u_{2rt}|^{r_2-1}u_{2rt}=g_2\left(u_{1r},u_{2r}\right) \end{array}\right\}, \tag{51}$$

$$D_{x_k}^{\beta_k}u_{1r}\left(t,x_k\left(0\right)\right)=D_{x_k}^{\beta_k}u_{1r}\left(t,x_k\left(1\right)\right)=0,\quad \beta_k=0,1,\ldots,\Lambda_{ik}-1, i=1,2,$$
$$k=1,...,n,\quad r=1,2,..., \tag{52}$$

$$u_{ir}\left(0,x\right)=\phi_{ir}\left(x\right),u_{irt}\left(0,x\right)=\psi_{ir}\left(x\right),\quad x\in\Pi_3, i=1,2,\quad r=1,2,... \tag{53}$$

are satisfied. In addition, the following a priori estimate is true

$$\sum_{i=1}^2\left[\left\|u_{irt}\left(t,\cdot\right)\right\|^2+\left\|\sum_{k=1}^3 D_{x_k}^{L_{ik}}u_{ir}(t,x)\right\|^2\right]\le c_r, 0\le t\le T_r, \tag{54}$$

$$\int_0^t\int_{\Pi_3}|u_{ir_s}|^{r_i+1}dxds\le c_r, \tag{55}$$

where $c_r=c\left(\sum_{i=1}^2\left[\left\|\sum_{k=1}^3 D_{x_k}^{\Lambda_{ik}}\phi_{ir}\right\|^2+\|\psi_{ir}\|^2\right]\right)$,

$$T_r=\frac{1}{2\left(p-1\right)\left(\sum_{i=1}^2\left[\left\|\sum_{k=1}^3 D_{x_k}^{\Lambda_{ik}}\phi_{ir}\right\|^2+\|\psi_{ir}\|^2\right]+1\right)}. \tag{56}$$

By virtue of (50)

$$c_r\le c,\quad r=1,2,..., \tag{57}$$

where c_r depends only on the expression $\sum_{i=1}^2\left[\left\|\sum_{k=1}^3 D_{x_k}^{ik}\phi_{ir}\right\|^2+\|\psi_{ir}\|^2\right]$.
By virtue of (56) and (57) there exists $N_0\in\{1,2,...\}$ such that for $r\ge N_0$ the following inequalities hold

$$T_r\ge T=\frac{1}{4\left(p-1\right)\left(\sum_{i=1}^2\left[\sum_{i=1}^2\left[\left\|\sum_{k=1}^3 D_{x_k}^{\Lambda_{ik}}\phi_i\right\|^2+\|\psi_i\|^2\right]+1\right]+1\right)}.$$

Hence the sequence $\{u_{ir}(t,\cdot), u_{ir_t}(t,\cdot)\}$ is bounded in the space

$$L_\infty\left(0,T;\hat{W}_2^{\overline{\Lambda}_i}\right)\times L_\infty\left(0,T;L_2\left(\Pi_3\right)\right).$$

Then from this sequence, we can choose subsequence which we will again denote by $\{u_{1r}(t,\cdot), u_{2r}(t,\cdot)\}$, such that as $k\to\infty$

$$u_{ir}\to u_i \quad \text{*-weakly in } L_\infty\left(0,T;\hat{W}_2^{\overline{\Lambda}_i}\right), \quad i=1,2; \tag{58}$$

$$u_{ir_t}\to u_{it} \quad \text{*-weakly in } L_\infty\left(0,T;L_2\left(\Pi_3\right)\right), i=1,2; \tag{59}$$

$$u_{ik_t}\to u_{it} \quad \text{*-weakly in } L_{r_i+1}\left(\left[0,T\right]\times\Pi_3\right), i=1,2. \tag{60}$$

From (58) and (59) it follows that

$$u_{ik}\to u_i \text{ in } C\left(\left[0,T\right];L_2\left(\Pi_3\right)\right), i=1,2. \tag{61}$$

Let us investigate whether the function $g_i(u_{1r},u_{2r})$ is converted to the function $g_i(u_1,u_2)$, $i=1,2$.

Using Lagrange's Mean Value Theorem, we obtain that

$$J_k=\left\|g_1\left(u_{1r},u_{2r}\right)-g_1\left(u_1,u_2\right)\right\|^2=$$

$$=\int_{\Pi_3}\left|\int_0^1\left(g_{1u_1}\left(u_1+\tau\left(u_{1r}-u_1\right),u_2+\tau\left(u_{2r}-u_2\right)\right)\left(u_{1r}-u_1\right)+\right.\right.$$
$$\left.+g_{2u_2}\left(u_1+\tau\left(u_{1r}-u_1\right),u_2+\tau\left(u_{2r}-u_2\right)\right)\left(u_{2r}-u_2\right)\right)d\tau|^2dx.$$

According to the embedding theorem, the following relations are true.

$$0\le J_k\le \sup_{x\in\overline{\Pi}_3}\left|g_{1u_1}\left(u_1+\tau\left(u_{1r}-u_1\right),u_2+\tau\left(u_{2r}-u_2\right)\right)\right|^2\left\|u_{1r}-u_1\right\|^2+$$

$$+\sup_{x\in\overline{\Pi}_3}\left|g_{1u_2}\left(u_1+\tau\left(u_{1r}-u_1\right),u_2+\tau\left(u_{2r}-u_2\right)\right)\right|^2\left\|u_{2r}-u_2\right\|^2\le$$

$$\le c\left(\left\|u_1\right\|_{C\left(\overline{\Pi}_n\right)},\left\|u_2\right\|_{C\left(\overline{\Pi}_n\right)}\right)\left[\left\|u_{1r}-u_1\right\|^2+\left\|u_{2r}-u_2\right\|^2\right]\le$$

$$\le c\left(\left\|u_1\right\|_{\hat{W}_2^{\overline{\Lambda}_1}},\left\|u_2\right\|_{\hat{W}_2^{\overline{\Lambda}_2}}\right)\left[\left\|u_{1r}-u_1\right\|^2+\left\|u_{2r}-u_2\right\|^2\right].$$

Then it follows from (61) that

$$\lim_{k\to\infty}J_k=0. \tag{62}$$

Thus, according to the relations (58) - (62), if we pass to the limit in the equation (51), we will get that , $(u_1,\ u_2)$ satisfies the problem (1) - (3), so that

$$u_i(\cdot)\in L_\infty\left(0,T;\hat{W}_2^{\overline{L}_i}\right), u_{it}(\cdot)\in L_\infty\left(0,T;L_2\left(\Pi_3\right)\right)\cap L_{r_i+1}\left(\left(0,T\right)\times\Pi_3\right), \quad i=1,2,$$

It follows that $h_i(t,x) = g_1(u_1,u_2) - |u_{1t}|^{r_1-1} u_{1t} \in L_2((0,T)\times \Pi_3)$, $i=1,2$.
It is obvious that the functions u_1, u_2 are a solution of the mixed problem

$$u_{itt} + \sum_{k=1}^{3} (-1)^{\Lambda_{ik}} D_{x_k}^{2\Lambda_{ik}} u_i = h_i(t,x),$$

$$D_{x_k}^{\beta_k} u_1 (t, x_k(0)) = D_{x_k}^{\beta_k} u_1 (t, x_k(1)) = 0, \quad \beta_k = 0,1,...,\Lambda_{ik}-1, i=1,2, \quad k=1,...,n,$$

$$u_i(0,x) = \phi_i(x), u_{it}(0,x) = \psi_i(x), \quad x \in \Pi_3, i=1,2.$$

It is known that if the solutions of the problem (1) - (3) satisfy the condition

$$u_i(\cdot) \in L_\infty\left(0,T; \hat{W}_2^{\overline{\Lambda}_i}\right), \quad u_{it}(\cdot) \in L_\infty(0,T; L_2(\Pi_3)), i=1,2,$$

then

$$u_i(\cdot) \in C\left([0,T]; \hat{W}_2^{\overline{\Lambda}_i}\right), \quad u_{it}(\cdot) \in C^1([0,T]; L_2(\Pi_3)), i=1,2.$$

(see [4, 7]).

5. The existence of a global solution

In some cases, for any $T>0$, the local solutions defined by Theorem 1 can be distributed over the entire $[0,T]\times\Pi_3$ region. According to Theorem 1, this is possible if the following a priori estimate is true for local solutions.

$$\sum_{i=1}^{2}\left[\|u_{it}(t,x)\|^2 + \left\|\sum_{k=1}^{3} D_{x_k}^{\Lambda_{ik}} u_i(t,x)\right\|^2\right] \le c, 0 \le t \le T. \tag{63}$$

We get this estimate if

$$\lambda = \frac{a_1(p_1+1)}{b_1} = \frac{a_2(p_2+1)}{b_2}, \tag{64}$$

$$a_i \le 0, b_i \le 0, \quad i=1,2. \tag{65}$$

Theorem 4. *Suppose that conditions* (4), (12), (64) *and* (65) *are satisfied, then for any* $T>0$, $(\phi_1,\phi_2) \in \hat{W}_2^{\overline{\Lambda}_1} \times \hat{W}_2^{\overline{\Lambda}_2}$ *and* $(\psi_1,\psi_2) \in L_2(\Pi_3)\times L_2(\Pi_3)$ *the problem* (1) - (3) *has a unique solution* $(u_1(\cdot), u_2(\cdot)) \in C\left([0,T]; \hat{W}_2^{\overline{\Lambda}_1} \times \hat{W}_2^{\overline{\Lambda}_2}\right) \cap C^1([0,T]; L_2(\Pi_3)\times L_2(\Pi_3))$.

Proof of the Theorem 4. Assume that $(u_1(\cdot), u_2(\cdot))$ is a local solution of the problem (1)-(3) in the domain $[0,T_{max}]\times\Pi_3$ defined by Theorem 2. Denote $b_i' = -b_i$, $i=1,2$, and multiply both sides of equation (1) by the function $\frac{p_i+1}{b'_i} u_{it}(t,x)$.

Integrating the resulting equality over the area $[0,T]\times\Pi_3$, we obtain

$$\frac{p_i+1}{b'_i}\int_0^t \int_{\Pi_3} u_{i_{ss}}(s,x)\, u_{i_s}(s,x)\, dxds +$$

$$+\frac{p_i+1}{b'_i}\int_0^t\int_{\Pi_3}\sum_{k=1}^{3}(-1)^{\Lambda_{ik}}D_{x_k}^{2\Lambda_{ik}}u_i(s,x)\,u_{i_s}(s,x)\,dxds+$$

$$+\frac{a_i(p_i+1)}{b'_i}\int_0^t\int_{\Pi_3}|u_{i_s}(s,x)|^{r_i+1}dxds=$$

$$=\frac{p_i+1}{b'_i}\int_0^t\int_{\Pi_3}g_i(u_1(s,x),u_2(s,x))\,u_{i_s}(s,x)\,dxds,$$

if we use integration by parts and sum the resulting equalities, we get the following:

$$\sum_{i=1}^{2}\frac{p_i+1}{2b'_i}\left[\int_{\Pi_3}|u_{i_t}(t,x)|^2dx+\int_{\Pi_3}\left|D_{x_k}^{\Lambda_{ik}}u_i(s,x)\right|^2dx+\right.$$

$$\left.+\,2\int_0^t\int_{\Pi_3}|u_{i_s}(s,x)|^{r_i+1}dxds\right]+$$

$$+\sum_{i=1}^{2}\frac{p_i+1}{b_i}\int_0^t\int_{\Pi_3}g_i(u_1(s,x),u_2(s,x))\,u_{i_s}(s,x)\,dxds=$$

$$=\sum_{i=1}^{2}\frac{p_i+1}{2b'_i}\left[\int_{\Pi_3}|\psi_i(x)|^2dx+\sum_{k=1}^{n}\int_{\Pi}\left|D_{x_k}^{\Lambda_{ik}}\phi_i(x)\right|^2dt\right]. \tag{66}$$

On the other hand, if we use the expression of the functions $g_1(u_1,u_2)$,$g_2(u_1,u_2)$ and the condition (64) , we get that

$$\sum_{i=1}^{2}\frac{p_i+1}{b_i}\int_0^t\int_{\Pi_3}g_i(u_1(s,x),u_2(s,x))\,u_{i_s}(s,x)\,dxds=$$

$$=\frac{\lambda}{p_1+p_2+2}\int_{\Pi_3}|u_1+u_2|^{p_1+p_2+2}dx+\int_{\Pi_3}|u_1|^{p_1+1}|u_2|^{p_2+1}dx-$$

$$-\frac{\lambda}{p_1+p_2+2}\int_{\Pi_3}|\phi_1+\phi_2|^{p_1+p_2+2}dx-\int_{\Pi_3}|\phi_1|^{p_1+1}|\phi_2|^{p_2+1}dx. \tag{67}$$

Considering (65) and (67) in (66), we obtain the following:

$$\sum_{i=1}^{2}\frac{p_i+1}{2b'_i}\left[\int_{\Pi_3}|u_{i_t}(t,x)|^2dx+\int_{\Pi_3}\left|D_{x_k}^{\Lambda_{ik}}u_i(s,x)\right|^2dx+\right.$$

$$\left.+\,2\int_0^t\int_{\Pi_3}|u_{i_s}(s,x)|^{r_i+1}dxds+\frac{\lambda}{p_1+p_2+2}\int_{\Pi_3}|u_1+u_2|^{p_1+p_2+2}dx+\int_{\Pi_3}|u_1|^{p_1+1}|u_2|^{p_2+1}dx\right]=$$

$$=\sum_{i=1}^{2}\frac{p_i+1}{2b'_i}\left[\int_{\Pi_3}|\psi_i(x)|^2dx+\sum_{k=1}^{n}\int_{\Pi_3}\left|D_{x_k}^{\Lambda_{ik}}\phi_i(x)\right|^2dt\right]+$$

$$+\frac{\lambda}{p_1+p_2+2}\int_{\Pi_n}|\phi_1(x)+\phi_2(x)|^{p_1+p_2+2}dx+\int_{\Pi_3}|\phi_1(x)|^{p_1+1}|\phi_2(x)|^{p_2+1}dx.$$

From this we obtain the a prior estimate (1).

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