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Inverse Boundary Value Problem for a Third-Order Partial Differential Equation with an Additional Integral Condition

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Abstract. In the article the author analyses one inverse boundary problem for a partial differential equation of third order with an Additional Integral Condition. First, an original problem is reduced to the equivalent problem, the theorem of existence and uniqueness of solution is proved for the latter. Then, using these facts the author proves existence and uniqueness of classical solution of the original problem.

Key Words and Phrases: inverse problem, differential equations, existence, uniqueness, classical solution.

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1. Introduction

Inverse problems are an actively developing branch of modern mathematics. Recently, inverse problems have arisen in various fields of human activity, such as seismology, mineral exploration, biology, medicine, quality control of industrial products, etc., which puts them among the actual problems of modern mathematics. Various inverse problems for certain types of partial differential equations have been studied in many works.

Let us note here, first of all, the works of A.N. Tikhonov [1], M.M. Lavrentev [2, 3], V.K. Ivanov [4] and their students. More details about this can be found in the monograph by A.M. Denisov [5].

The purpose of this work is to prove the existence and uniqueness of solutions of one inverse boundary value problem for a third order differential equation with partial derivatives with an integral condition of the first kind.

In this work, using Fourier method and contraction mapping principle, we prove the existence and uniqueness of the solution of the nonlocal inverse boundary value problem for a third order two-dimensional pseudo parabolic equation.

2. Formulation of the inverse boundary value problem

Consider for the equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial}{\partial t}\left(a(t)\frac{\partial^2 u(x,t)}{\partial x^2}\right) = p(t)u(x,t) + f(x,t) \tag{1}$$

in the domain $D_T = \{(x,t) : 0 \le x \le 1,\ 0 \le t \le T\}$ inverse boundary value problem with initial conditions

$$u(x,0) = \varphi(x)\,,\ u_t(x,0)\ = \psi(x)\ \ (0 \le x \le 1)\ , \tag{2}$$

with Neumann boundary conditions

$$u_x(0,t) = u(1,t) = 0\ \ (0 \le t \le T)\,, \tag{3}$$

and with an additional integral condition

$$\int_0^1 g(x)u(x,t)dx = 0\ (0 \le t \le T) \tag{4}$$

where $a(t) > 0,\ f(x,t),\ \varphi(x),\ \psi(x),\ \omega(x)\ ,\ h(t)$ are given functions, and $u(x,t)$ and $p(t)$ are unknown functions.

Let us introduce the notation

$$\tilde{C}^{2,2}(D_T) =\ \{u(x,t) : u(x,t) \in C^2(D_T),\ u_{txx}(x,t) \in C(D_T)\}\,.$$

Definition 1. *Under the classical solution of the inverse boundary value problem* (1)-(4) *we mean a pair* $\{u(x,t), p(t)\}$ *of functions* $u(x,t),\ p(t)$ *, if* $u(x,t) \in \tilde{C}^{2,2}(D_T),\ p(t) \in C[0,T]$ *and the relations* (1)-(4) *are satisfied in the usual sense .*

The following theorem is true.

Theorem 1. *Let* $f(x,t) \in C(D_T),\ \ \psi(x) \in C[0,\ 1],\ \varphi(x) \in C[0,\ 1],\ h(t) \in C^2[0,T]$ *,* $0 < a(t) \in C^1[0,\ T],\ h(t) \neq 0\ \ (0 \le t \le T)$ *and the matching conditions are met*

$$\int_0^1 g(x)\varphi(x)dx = 0, \quad \int_0^1 g(x)\psi(x)dx = 0.$$

Then the problem of finding a classical solution to problem (1)-(4) *is equivalent to the problem of determining functions* $u(x,t) \in \tilde{C}^{2,2}(D_T),\ p(t) \in C[0,T]$*, satisfying equation* (1)*, conditions* (2)*,* (3) *and conditions*

$$h''(t) - \frac{d}{dt}\left(a(t)\int_0^1 g(x)\frac{\partial^2 u(x,t)}{\partial x^2}dx\right) = p(t)h(t) + \int_0^1 g(x)f(x,t)dx\ (0 \le t \le T). \tag{5}$$

Proof. Let $\{u(x,t), p(t)\}$ be a classical solution to problem (1)-(4). Since $h(t) \in C^2[0,T]$, we differentiate (4) twice with respect to t, we get:

$$\int_0^1 g(x)u_t(x,t)dx = h'(t)\ ,\quad \int_0^1 g(x)u_{tt}(x,t)dx = h''(t) \quad (0 \le t \le T). \tag{6}$$

We multiply equation (1) by the function $g(x)$ and integrate the resulting equality from 0 to 1 with respect to x, we have:

$$\frac{d^2}{dt^2}\int_0^1 g(x)u(x,t)dx - \frac{d}{dt}\left(a(t)\int_0^1 g(x)\frac{\partial^2 u(x,t)}{\partial x^2}dx\right) =$$

$$= p(t)\int_0^1 g(x)u(x,t)dx + \int_0^1 g(x)f(x,t)dx \ (\ 0 \le t \le T\,). \tag{7}$$

Hence, taking into account (4) and (6), we easily arrive at the fulfillment of (5).

Now, suppose that $\{u(x,t), p(t)\}$ is a solution to problem (1)-(3), (5).

Then from (5) and (7) we get:

$$\frac{d^2}{dt^2}\int_0^1 g(x)u(x,t)dx = p(t)\int_0^1 g(x)u(x,t)dx \ (0 \le t \le T). \tag{8}$$

Due to (2) and $\int_0^1 g(x)\varphi(x)dx = 0$, $\int_0^1 g(x)\psi(x)dx = 0$ it is clear that

$$\int_0^1 g(x)u(x,0)dx = \int_0^1 g(x)\varphi(x)dx = 0, \int_0^1 g(x)u_t(x,0)dx = \int_0^1 g(x)\psi(x)dx = 0. \tag{9}$$

From (8) and (9) we conclude that condition (4) is satisfied. Theorem is proved.

3. On the solvability of the inverse boundary value problem

The first component $u(x,t)$ of the solution $\{u(x,t), p(t)\}$ of problem (1)-(3), (5) will be sought in the form:

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t)\cos\lambda_k x \quad \left(\lambda_k = \frac{\pi}{2}(2k-1)\right), \tag{10}$$

where

$$u_k(t) = 2\int_0^1 u(x,t)\cos\lambda_k x dx \ \ (k = 1, 2, ...).$$

Then, applying the formal scheme of the Fourier method, from (1), (2), we obtain:

$$u_k''(t) + \lambda_k^2(a(t)u_k(t))' = F_k(t; u, p) \ \ (k = 1, 2, ...;\ 0 \le t \le T), \tag{11}$$

$$u_k(0) = \varphi_k, u_k'(0) = \psi_k \ (k = 1, 2, ...), \tag{12}$$

where

$$F_k(t; u, p) = f_k(t) + p(t)u_k(t), f_k(t) = 2\int_0^1 f(x,t)\cos\lambda_k x dx, ,$$

$$\varphi_k = 2\int_0^1 \varphi(x)\cos\lambda_k x dx, \psi_k = 2\int_0^1 \psi(x)\cos\ \lambda_k x dx \ \ (k = 1, 2, ...).$$

Solving problem (11), (12) we find:

$$u_k(t)=\varphi_k\left(e^{-\lambda_k^2\int_0^t a(s)ds}+\lambda_k^2 a(0)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)+\psi_k\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau+$$

$$+\int_0^t F_k(\eta;u,p)\left(\int_\eta^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)d\eta \quad (k=1,2,...). \tag{13}$$

Differentiating twice (21) we obtain:

$$u_k'(t)=-\lambda_k^2\varphi_k\left(a(t)e^{-\lambda_k^2\int_0^t a(s)ds}-a(0)\left(1-\lambda_k^2 a(t)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)\right)+$$

$$+\psi_k\left(1-\lambda_k^2 a(t)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)+$$

$$+\int_0^t F_k(\eta;u,p,q)\left(1-\lambda_k^2 a(t)\int_\eta^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)d\eta \quad (k=1,2,...), \tag{14}$$

$$u_k''(t)=-\lambda_k^2\varphi_k\left((a'(t)-\lambda_k^2 a^2(t))e^{-\lambda_k^2\int_0^t a(s)ds}+\lambda_k^2 a(0)(a'(t)-\right.$$

$$\left.-\lambda_k^2 a^2(t))\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)-\lambda_k^2\psi_k(a'(t)-\lambda_k^2 a^2(t))\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau-$$

$$-\lambda_k^2\int_0^t F_k(\eta;u,p)\left((a'(t)-\lambda_k^2 a^2(t))\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau+a(t)\right)d\eta+$$

$$+F_k(t;u,p)\,(k=1,2,...). \tag{15}$$

After substituting the expression $u_k(t)$ $(k=1,2,...)$ from (13) into (10), to determine the the component $u\,(x,t)$ of the solution of problem (5) we obtain:

$$u(x,t)=\sum_{k=1}^{\infty}\left\{\varphi_k\left(e^{-\lambda_k^2\int_0^t a(s)ds}+\lambda_k^2 a(0)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)+\right.$$

$$\left.+\psi_k\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau+\int_0^t F_k(\eta;u,p)\left(\int_\eta^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)d\eta\right\}\cos\lambda_k x\,. \tag{16}$$

Now from (5), taking into account (10), we get:

$$p(t)=[h(t)]^{-1}\left\{h''(t)-\int_0^1 g(x)f(x,t)dx+\sum_{k=1}^{\infty}n_k\lambda_k^2(a(t)u_k(t))'\right\}, \tag{17}$$

where

$$n_k=\int_0^1 g(x)\cos\lambda_k x dx. \tag{18}$$

Further, from (18), by virtue of (25) we find:

$$\lambda_k^2(a(t)u_k(t))' = -u_k''(t) + F_k(t;u,p,q) =$$

$$= \lambda_k^2\varphi_k\left((a'(t)-\lambda_k^2a^2(t))\left(e^{-\lambda_k^2\int_0^t a(s)ds} + \lambda_k^2 a(0)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)\right)+$$

$$+\lambda_k^2\psi_k(a'(t)-\lambda_k^2a^2(t))\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau+$$

$$+\lambda_k^2\int_0^t F_k(\eta;u,p)\left((a'(t)-\lambda_k^2a^2(t))\int_\eta^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau + a(t)\right)d\eta\,(k=1,2,\dots) \qquad (19)$$

In order to obtain an equation for the second component $p(t)$ of the solution $\{u(x,t),p(t)\}$ of problem (1)-(3), (5) we substitute the expression $\lambda_k^2(a(t)u_k(t))'\,(k=1,2,...)$ from (19) to (17) . We have:

$$p(t) = [h(t)]^{-1}\left\{h''(t) - \int_0^1 g(x)f(x,t)dx + \right.$$

$$+\sum_{k=1}^{\infty} n_k\left[\lambda_k^2\varphi_k\left((a'(t)-\lambda_k^2a^2(t))\left(e^{-\lambda_k^2\int_0^t a(s)ds} + \lambda_k^2 a(0)\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau\right)\right)+\right.$$

$$+\lambda_k^2\psi_k(a'(t)-\lambda_k^2a^2(t))\int_0^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau+$$

$$\left.\left.+\lambda_k^2\int_0^t F_k(\eta;u,p)\left((a'(t)-\lambda_k^2a^2(t))\int_\eta^t e^{-\lambda_k^2\int_\tau^t a(s)ds}d\tau + a(t)\right)d\eta\right]\right\}, \qquad (20)$$

Thus, the solution of problem (1)-(3), (5) is reduced to the solution of system (16), (20) with respect to unknown functions $u(x,t)$ and $p(t)$.

To study the question of the uniqueness of the solution of problem (1) - (3), (5), the following lemma plays an important role.

Lemma 1. *If* $\{u(x,t),p(t)\}$ *- be any solution to the problem* (1)-(3), (5), *then the functions*

$$u_k(t) = 2\int_0^1 u(x,t)\cos\lambda_k x dx \ \ (k=1,2,...)$$

satisfy the system consisting of equations (13).

It is obvious that if $u_k(t) = 2\int_0^1 u(x,t)\cos\lambda_k xdx(k=1,2,...)$ is a solution to system (20) and (21), then the pair $\{u(x,t),p(t)\}$ of a function $u(x,t)=\sum_{k=0}^{\infty}u_k(t)\cos\lambda_k x$ and $p(t)$ are solutions to system (16), (20) .

It follows from Lemma 1 that the following corollary holds.

Corollary 1. *Let system* (16), (20) *have a unique solution. Then problem* (1)-(3), (5) *cannot have more than one solution, i.e. if problem* (1)-(3), (5) *has a solution, then it is unique.*

1. Denote by $B_{2,T}^3$, the set of all functions $u(x,t)$ of the form

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t) \cos \lambda_k x \ ,$$

considered in D_T, where each of the functions $u_k(t)\,(k = 1, 2, ...)$ is continuous on $[0, T]$ and

$$I(u) \equiv \left\{ \sum_{k=1}^{\infty} (\lambda_k^3 \, \|u_k(t)\|_{C[0,T]})^2 \right\}^{\frac{1}{2}} < +\infty.$$

We define the norm on this set as follows:

$$\|u(x,t)\|_{B_{2,T}^3} = I(u).$$

2. Denote by E_T^3 the space consisting of the topological product

$$B_{2,T}^3 \times C[0,T]\,.$$

The norm of an element $z = \{u, p\}$ is defined by the formula

$$\|z\|_{E_T^3} = \|u(x,t)\|_{B_{2,T}^3} + \|p(t)\|_{C[0,T]}\,.$$

It is known that $B_{2,T}^3$ and E_T^3 are Banach spaces.
Now consider in space E_T^3 the operator

$$\Phi(u,a) = \{\Phi_1(u,p), \Phi_2(u,p)\} \ ,$$

where

$$\Phi_1(u,p) = \tilde{u}(x,t) = 1 \sum_{k=0}^{\infty} \tilde{u}_k(t) \cos \lambda_k x, \Phi_2(u,p) = \tilde{p}(t).$$

and $\tilde{u}_k(t)$ and $\tilde{p}(t)$ are equal to the right-hand sides of (13) and (20), respectively.
It is easy to see that

$$\int_0^t e^{-\lambda_k^2 \int_\tau^t a(s)ds} d\tau \le \frac{1}{m\lambda_k^2}, \quad \int_\eta^t e^{-\lambda_k^2 \int_\tau^t a(s)ds} d\tau \le \frac{1}{m\lambda_k^2},$$

where $m = \min\limits_{0 \le t \le T} a(t)$.
Considering these relations, we find:

$$\left(\sum_{k=1}^{\infty} \left(\lambda_k^3 \, \|\tilde{u}_k(t)\|_{C[0,T]} \right)^2 \right)^{\frac{1}{2}} \le 2 \left(1 + \frac{a(0)}{m} \right) \left(\sum_{k=1}^{\infty} (\lambda_k^3 \ \ |\varphi_k|)^2 \right)^{\frac{1}{2}} +$$

$$+ \frac{2}{m} \left(\sum_{k=1}^{\infty} (\lambda_k \ \ |\psi_k|)^2 \right)^{\frac{1}{2}} + \frac{2\sqrt{T}}{m} \left(\int_0^T \sum_{k=1}^{\infty} (\lambda_k \ |f_k(\tau)|)^2 \, d\tau \right)^{\frac{1}{2}} +$$

$$+\frac{2T}{m}\left\|p(t)\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\left(\lambda_k^2\ \left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}, \tag{21}$$

$$\left\|\tilde{p}(t)\right\|_{C[0,T]}\leq\left\|[h(t)]^{-1}\right\|_{C[0,T]}\left\{\left\|h''(t)-\int_0^1 g(x)f(x,t)dx\right\|_{C[0,T]}+\right.$$

$$+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left(\left\|a'(t)\right\|_{C[0,T]}+\left\|a^2(t)\right\|_{C[0,T]}\right)\times$$

$$\times\left\|g(x)\right\|_{L_2(0,1)}\left[\left(1+\frac{a(0)}{m}\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^5\ \left|\varphi_k\right|\right)^2\right)^{\frac{1}{2}}+\frac{1}{m}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\ \left|\psi_k\right|\right)^2\right)^{\frac{1}{2}}+\right.$$

$$\left.\left.+\frac{\sqrt{T}}{m}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k^3\ \left|f_k(\tau)\right|\right)^2d\tau\right)^{\frac{1}{2}}+\frac{T}{m}\left\|p(t)\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\ \left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{\frac{1}{2}}\right]\right\}, \tag{22}$$

Let us assume that the data of problem (1)-(3), (5) satisfy the following conditions:
1) $\varphi(x)\in C^4[0,1],\varphi^{(5)}(x)\in L_2(0,1)$,

$$\varphi'(0)=\varphi(1)=\varphi'''(0)=\varphi''(1)=\varphi^{(4)}(1)=0\ ,$$

2) $\psi(x)\in C^2[0,1],\psi'''(x)\in L_2(0,1),\ \psi'(0)=\psi(1)=\psi''(1)=0$.
3) $f(x,t),f_x(x,t),\ f_{xx}(x,t)\in C(D_T),f_{xxx}(x,t)\in L_2(D_T)$,

$$f_x(0,t)=f(1,t)=f_{xx}(1,t)=0\ \ (0\leq t\leq T),$$

4) $b(x)\in L_2(0,1),0<a(t)\in C^1[0,T],\ h(t)\in C^2[0,T]\ ,\ h(t)\neq 0\,(0\leq t\leq T)$.
Then from (21) , (22), we get:

$$\left\|\tilde{u}(x,t)\right\|_{B^3_{2,T}}\ \leq A_1(T)+B_1(T)\left\|p(t)\right\|_{C[0,T]}\left\|u(x,t)\right\|_{B^3_{2,T}}\ , \tag{23}$$

$$\left\|\tilde{p}(t)\right\|_{C[0,T]}\ \leq A_2(T)+B_2(T)\left\|p(t)\right\|_{C[0,T]}\left\|u(x,t)\right\|_{B^3_{2,T}}\,. \tag{24}$$

where

$$A_1(T)=\left\|\varphi(x)\right\|_{L_2(0,1)}+T\left\|\psi(x)\right\|_{L_2(0,1)}+T\sqrt{T}\left\|f(x,t)\right\|_{L_2(D_T)}+$$

$$A_1(T)=2\left(1+\frac{a(0)}{m}\right)\left\|\varphi'''(x)\right\|_{L_2(0,1)}+\frac{2}{m}\left\|\psi'(x)\right\|_{L_2(0,1)}+\frac{2\sqrt{T}}{m}\left\|f_x(x,t)\right\|_{L_2(D_T)},$$

$$B_1(T)=\frac{2T}{m},$$

$$A_2(T)=\left\|[h(t)]^{-1}\right\|_{C[0,T]}\left\{\left\|h''(t)-\int_0^1 g(x)f(x,t)dx\right\|_{C[0,T]}+\right.$$

$$+\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left(\left\|a'(t)\right\|_{C[0,T]}+\left\|a^2(t)\right\|_{C[0,T]}\right)\|g(x)\|_{L_2(0,1)}\times$$

$$\left.\times\left[\left(1+\frac{a(0)}{m}\right)\left\|\varphi^{(5)}(x)\right\|_{L_2(0,1)}+\frac{1}{m}\left\|\psi'''(x)\right\|_{L_2(0,1)}+\frac{\sqrt{T}}{m}\left\|f_{xxx}(x,t)\right\|_{L_2(D_T)}\right]\right\}$$

$$B_2(T)=\left\|[h(t)]^{-1}\right\|_{C[0,T]}\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left(\left\|a'(t)\right\|_{C[0,T]}+\left\|a^2(t)\right\|_{C[0,T]}\right)\frac{T}{m}.$$

From inequalities (23), (24) we conclude:

$$\|\tilde{u}(x,t)\|_{B^3_{2,T}}+\|\tilde{p}(t)\|_{C[0,T]}\le A(T)+B(T)\,\|p(t)\|_{C[0,T]}\,\|u(x,t)\|_{B^3_{2,T}}. \qquad (25)$$

where

$$A(T)=A_1(T)+A_2(T),\quad B(T)=B_1(T)+B_2(T).$$

So, we can prove the following theorem:

Theorem 2. *Let conditions 1)-4) be satisfied and*

$$B(T)(A(T)+2)^2<1\ , \qquad (26)$$

Then problem (1)-(3), (5) *has a unique solution in the ball* $K=K_R(\|z\|_{E^3_T}\le R= = A(T)+2)$ *of space* E^3_T.

Proof. In space E^3_T consider the following equation

$$z=\Phi z, \qquad (27)$$

where $z=\{u,p\}$, components $\Phi_i(u,p)(i=1,2)$ of the operator (u,p) are defined by the right-hand sides of equations (16), (20), respectively. Consider the operator $\Phi(u,p)$ in the ball $K=K_R(\|z\|_{E^3_T}\le R=A(T)+2)$ from E^3_T.

Similarly to (23), we obtain that for any $z,z_1,z_2\in K_R$ the estimates

$$\|\Phi z\|_{E^3_T}\le A(T)+B(T)\,\|p(t)\|_{C[0,T]}\,\|u(x,t)\|_{B^3_{2,T}}\le A(T)+B(T)(A(T)+2)^2\ , \qquad (28)$$

$$\|\Phi z_1-\Phi z_2\|_{E^3_T}\le\le B(T)R\,\left(\|u_1(x,t)-u_2(x,t)\|_{B^3_{2,T}}+\|p_1(t)-p_2(t)\|_{C[0,T]}\right). \qquad (29)$$

are valid. Then from the estimates (28) and (29), taking into account (26), it follows that the operator Φ acts in the ball $K=K_R$and is contractive. Therefore, in the ball $K=K_R$ the operator Φ has a unique fixed point $\{u,p\}$, which is the only solution of the equation (27), i.e. is the unique solution in the ball $K=K_R$ of the system. The function $u(x,t)$, as an element of space $B^3_{2,T}$, is continuous and has continuous derivatives $u_x(x,t)$, $u_{xx}(x,t)$, in D_T.

It can be shown that $u_{tt}(x,t)$,$u_{txx}(x,t)$, are continuous in D_T.

It is easy to check that equation (1) and conditions (2), (3), and (5) are satisfied in the usual sense. Consequently, $\{u(x,t), p(t)\}$ is a solution to problem (1)-(3), (5), and, by virtue of the corollary of Lemma 1, it is unique in the ball $K = K_R$. Theorem is proved.

Using Theorem 1, we prove the following.

Theorem 3. *Let all conditions of Theorem 2 be satisfied and*

$$\int_0^1 g(x)\varphi(x)dx = h(0)\,, \quad \int_0^1 g(x)\psi(x)dx = h'(0)\ .$$

Then problem (1)-(4) *has a unique classical solution in the ball* $K = K_R(\|z\|_{E_T^3} \le R = A(T)+2)$ *of space* E_T^3.

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