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Problem Statement about the Unsymmetrical Oscillations of a Cylindrical cover Reinforced with rods Subjected to a Compressive Force Along the Axis with a Fluid

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Abstract. In the article, a physical and mathematical model was built to study the issue of asymmetric oscillations of a cylindrical cover reinforced with rods under the influence of a compressive force in the direction of the axis, together with a liquid. The frequency equation of the system was established and calculated by the asymptotic method. effect has been studied. Problem statement about the unsymmetrical oscillations of a cylindrical cover reinforced with rods subjected to a compressive force along the axis with a fluid.

Key Words and Phrases: compressive force, reinforced with rods, cylindrical cover, dances.
2010 Mathematics Subject Classifications: Primary 34A55, 34B20, 34L05

1. Introduction

A rod-reinforced cylindrical cover is a structural element consisting of a combination of cover and rods that deforms together. It is assumed that the cover and the rods interact along a certain line and the conditions of equality of displacements on their contact line are satisfied. This method was used in [1] to obtain the equilibrium equations and natural boundary conditions of a cylindrical cover reinforced with rods. Equilibrium equations for a cover reinforced with rods were obtained in [2]. The system of equilibrium equations of a cover with rods in an arbitrary position on its surface was obtained in works [3].

2. Problem solving method

The system we studied consists of a cylindrical cover reinforced with rods and a liquid that completely fills its interior. Therefore, in order to study the oscillations of such a system, we will use the system of equations of motion of a cylindrical cover reinforced with rods and the contact conditions added to them.

A rod-reinforced cylindrical cover means a cylindrical cover and a system consisting of rods rigidly attached to it along the coordinate lines (Fig. 1). It is assumed that the coordinate axes coincide with the main curvature lines of the coating, and the rods are in rigid contact with the coating along these lines. Using the Ostrogradsky-Hamilton variation principle, the system of basic equations of the mentioned system can be deduced. It is considered that the stress-strain state of the cylindrical cover is completely determined by the equations of the linear theory of covers based on the Kirchhoff-Liav hypothesis. In the calculation of rods, equations based on the Kirchhoff-Klebsch theory are used for straight-axis rods. If the cylindrical coating is under the influence of compressive stress along the axis, its potential energy is determined as follows [2]:

$$\Psi = \frac{Eh}{2\left(1-\nu^2\right)}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial u}{\partial\xi}+\frac{\partial v}{\partial\theta}-w\right)^2+2\left(1-\nu\right)\left[\frac{\partial u}{\partial\xi}\left(\frac{\partial v}{\partial\theta}-w\right)-\right.\right.$$

$$\left.\left.-\frac{1}{4}\left(\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\right\}d\xi d\theta+\frac{Eh^3}{24\left(1-\nu^2\right)R^2}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)^2-\right.$$

$$\left.-2\left(1-\nu\right)\left[\frac{\partial^2 w}{\partial\xi^2}\left(\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)-\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\right\}d\xi d\theta+$$

$$+\frac{E_s}{2\mathrm{R}}\sum_{j=1}^{k_1}\int_0^{2\pi}\left[F_s\left(\frac{\partial v}{\partial\theta}-w-\frac{h_s}{R}\frac{\partial^2 w}{\partial\theta^2}\right)^2+\frac{I_{xs}}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2+\frac{G_s}{R^2E_s}I_{kp.s}\times\right. \tag{1}$$

$$\left.\times\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial u}{\partial\theta}\right)^2\right]\Bigg|_{\xi=\xi_j}d\theta-\frac{\sigma_x h}{2}\int_0^{\xi_1}\int_0^{2\pi}\left(\frac{\partial w}{\partial\xi}\right)^2 d\xi d\theta-\frac{\sigma_x F_c}{2R}\sum_{i=1}^{k}\int_0^{\xi_1}\left(\frac{\partial w}{\partial\xi}\right)^2\Bigg|_{\theta=\theta_i}d\xi+$$

$$+\frac{E_c}{2R}\sum_{i=1}^{k_2}\int_0^{\xi_1}\left[F_c\left(\frac{\partial u}{\partial\xi}-\frac{h_c}{R}\frac{\partial^2 w}{\partial\xi^2}\right)^2+\frac{I_{us}}{R^2}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2+\frac{G_c}{E_c}I_{kp.s}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\Bigg|_{\theta=\theta_i}d\xi\ .$$

In his expressions u, v, w - cover displacements, $E, \nu-$ respectively, the modulus of elasticity and Poisson's ratio of the material of the cylindrical coating, R, h- respectively, the radius and thickness of the cylindrical coating, E_c, E_s- the modulus of elasticity of the longitudinal bar and the ring, respectively, F_c, F_s - the cross-sectional areas of the longitudinal bar and the ring, respectively, I_{us}, $I_{kp.s}$ - moments of inertia of the cross section of the longitudinal bar, $I_{xs}, I_{kp.s}$ - moments of inertia of the cross section of the ring, q_x, q_θ, q_r - components of the pressure force acting on the cylindrical cover by the medium, G_c, G_s - are the shear modulus of the longitudinal bar and the ring, respectively.

$$K=\frac{Eh}{2(1-\nu^2)}\int_0^{\xi_1}\int_0^{2\pi}\left[\left(\frac{\partial u}{\partial t_1}\right)^2+\left(\frac{\partial v}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]d\xi d\theta+$$

$$+\frac{\bar{\rho}_c E_c F_c}{2R\left(1-\nu^2\right)}\sum_{i=1}^{k_2}\int_0^{\xi_1}\left[\left(\frac{\partial u}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]\Bigg|_{\theta=\theta_j}d\xi+$$

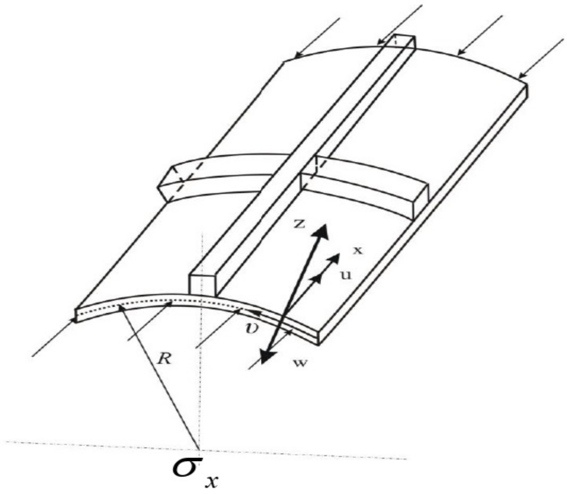

Fig 1. Cylindrical cover reinforced with rods subjected to compressive force and in contact with the environment. The kinetic energy of the rod-reinforced coating is as follo

$$+\frac{\bar{\rho}_s E_s F_s}{2R\left(1-\nu^2\right)}\sum_{i=1}^{k_1}\int_0^{2\pi}\left[\left(\frac{\partial v}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]\Bigg|_{\xi=\xi_j} d\theta \tag{2}$$

Using the decision condition of the Ostrogradsky-Hamilton effect, the equation of motion of the cover reinforced with rods can be obtained:

$$\delta W=\delta\left(\Psi+K\right)=0 \tag{3}$$

Expressions of potential and kinetic energy (1) and (2)- is also shown. Here $W=\int_{t_0}^{t_1}\tilde{L}dt$ Hamilton effect, $\tilde{L}=K-$. It is a lag function. (3) if we carry out the operation of taking variations in the equation and δu, δv, δw If we take into account that the variation is arbitrary and independent, we get the following system of equations of motion:

$$\begin{cases} \mathrm{L_x}(u,v,w)+q_x=0 \\ \mathrm{L_y}(u,v,w)+q_y=0 \\ \mathrm{L_z}(u,v,w)-\left(q_z-q_{zz}\right)=0 \end{cases} \tag{4}$$

The propagation of small excitations in an ideal fluid is expressed by the following equation.

$$\nabla^2\Phi-\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}=0. \tag{5}$$

Here Φ- potential of liq, a- is the speed of sound propagation in a liquid. In a harmonic dance (5) converts the equation to the Helmolts equation:

$$\nabla^2\Phi + \frac{\omega^2}{a^2}\Phi = 0. \tag{6}$$

When the fluid is incompressible $a^2 \to \infty$ since (6) transforms the equation into Laplace's equation:

$$\nabla^2\Phi = 0. \tag{7}$$

If the liquid is an ideal liquid with two-phase bubbles, the propagation of small perturbations in such a liquid is given by the following equation [2]:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{a^2}\frac{\partial^2 p}{\partial t^2} - \frac{2\rho_{j>}R}{Eh}\frac{\partial^2 p}{\partial t^2} = 0. \tag{8}$$

Here, $a^2 = \frac{1}{\alpha_{20}(1-\alpha_{20})}\left(\frac{\rho_{10}}{\rho_{10}-\rho_{20}}\right)\frac{p_0}{p_{10}^0}$, ρ_{10}, ρ_{20}- true density of liquid and gas; p_0- static pressure; ρ_{jo}- density of the mixture; α_{20}- volume of gas bubbles; The equilibrium values of the parameters correspond to the zero index;

$$\rho_{j>} = (1-\alpha_{20})\rho_{10} + \alpha_2\rho_{20}.$$

Contact conditions are also added to the system of equations of motion of the coating (3), equations of motion of the fluid (4), (6). The normal components of velocity and pressure on the contact surface of the coating with the liquid are assumed to be equal, and the tangential stresses are equal to zero:

$$\vartheta_r = \frac{\partial w}{\partial t}, \quad q_z = -p, \quad q_x = q_y = 0 \quad (r = R) \tag{9}$$

Here q_x, q_y, q_z are the components of the pressure force exerted by the fluid on the coating. The systems of equations of motion of the rod-reinforced coating and liquid (4)-(8) together with the contact conditions (9) allow solving the problem of free oscillations of the constructive-orthotropic coating-liquid system. In other words, the study of the free oscillations of an orthotropic cylindrical coating in contact with a solid medium and a liquid is brought to the joint integration of the system of equations of the constructive-orthotropic coating and the equation of motion of the liquid within the contact conditions.

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