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Global Bifurcation from Infinity of Nondifferentiable Perturbations of Half-linear Eigenvalue Problems for Ordinary Differential Equations of Fourth Order

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Abstract. In this paper, we consider the nondifferentiable perturbations of a half-linear eigenvalue problem for ordinary differential equations of fourth order. We show the existence of global components of a set of nontrivial solutions bifurcating from intervals containing half-eigenvalues of a half-linear eigenvalue problem and contained in classes of functions that have oscillatory properties of half-eigenfunctions of a half-linear eigenvalue problem and their derivatives in some neighborhoods of these intervals.

Key Words and Phrases: nondifferentiable perturbations, half-linear eigenvalue problem, half-eigenfunction, bifurcation interval, global component

2010 Mathematics Subject Classifications: 34B05, 34B24, 34C23, 34L20, 34l30, 34K18, 47J10, 47J15

1. Introduction

We consider the following nonlinear eigenvalue problem

$$\begin{aligned}\ell y \equiv (p\,(x)\,y'')'' - (q(x)y')' + r(x)y = \lambda\tau(x)\,y + \alpha(x)y^+(x) + \beta(x)y^-(x) + \\ f(x,y,y',y'',y''',\lambda) + g(x,y,y',y'',y''',\lambda),\ x\in(0,l),\end{aligned} \tag{1.1}$$

$$y(0) = y'(0) = y(l) = y'(l) = 0,, \tag{1.2}$$

where $\lambda\in\mathbb{R}$ is a parameter, $p\in C^2\left([0,l];(0,+\infty)\right)$, $q\in C^1\left([0,l];[0,+\infty)\right)$, $\tau\in C\left([0,l];(0,+\infty)\right)$, r, α, $\beta\in C\left([0,l];\mathbb{R}\right)$, and $y^+=\max\{y,0\}$, $y^-=(-y)^+$. The nonlinear terms f and g satisfy the following conditions: f, $g\in C\left([0,l]\times\mathbb{R}^5;\mathbb{R}\right)$; there exists constant $M>0$ such that

$$\left|\frac{f(x,y,s,v,w,\lambda)}{y}\right| \le M,\ x\in[0,l],\ (y,s,v,w)\in\mathbb{R}^4,\ y\neq 0,\ \lambda\in\mathbb{R}; \tag{1}$$

$$g(x,y,s,v,w,\lambda) = o(|y|+|s|+|v|+|w|) \text{ as } |y|+|s|+|v|+|w|\to+\infty, \tag{2}$$

uniformly in $x \in [0, l]$ and in $\lambda \in \Lambda$, for any bounded interval $\Lambda \subset \mathbb{R}$.

The global bifurcation of nontrivial solutions from zero and infinity to nonlinear Sturm-Liouville problems of second and fourth orders was studied in detail in [1, 3, 5, 10, 11, 13, 14] (see also references therein). These papers prove the existence of global continua of nontrivial solutions in $\mathbb{R} \times C^1$ and $\mathbb{R} \times C^3$, respectively, emanating from bifurcation points and bifurcation intervals (in $\mathbb{R} \times \{0\}$ and $\mathbb{R} \times \{\infty\}$, respectively, which we identify with $\mathbb{R}$), containing the eigenvalues of linear problems obtained from nonlinear problems by setting the nonlinear terms to zero. Moreover, it has been established that these global continua in the neighborhoods of bifurcation points and intervals are contained in classes of functions with fixed oscillation count.

Global bifurcation from zero and infinity in half-linearizable Sturm-Liouville problems of second order was studied in [5, 12, 15] (it should be noted that the nonlinear problems considered in [15] are nonlinear perturbations of linear eigenvalue problems for self-adjoint operators of $2m$th-order). These papers show the existence of global continua of nontrivial solutions in $\mathbb{R} \times C^1$, bifurcating from half-eigenvalues of half-linear eigenvalue problems and contained in classes of functions having the usual nodal properties. Similar results for half-linearizable Sturm-Liouville problems of fourth order were obtained in [2, 7, 8, 15].

In [9] we consider the nondifferentiable perturbations of a half-linear Sturm-Liouville problem of fourth order. We prove the existence of two families of global components of the set of nontrivial solutions in $\mathbb{R} \times C^3$, emanating from bifurcation intervals containing the half-eigenvalues of the corresponding half-linear eigenvalue problem and contained in classes of functions having oscillation properties of half-eigenfunctions (and their derivatives) of this half-linear eigenvalue problem.

In this paper, we consider global bifurcation from infinity of nonlinear eigenvalue problem (1.1), (1.2) under conditions (1.3) and (1.4). We prove the existence of two families unilateral global components of a set of nontrivial solutions bifurcating from intervals containing half-eigenvalues of a half-linear eigenvalue problem (1.1), (1.2) with $f, g \equiv 0$ and contained in classes of functions that have oscillatory properties of half-eigenfunctions of this half-linear eigenvalue problem and their derivatives in some neighborhoods of these intervals.

2. Preliminary

Let E be the Banach space

$$C^3[0, l] \cap \{y : y(0) = y'(0) = y(1) = y'(1) = 0\}$$

with the usual norm

$$||y||_3 = ||y||_\infty + ||y'||_\infty + ||y''||_\infty + ||y'''||_\infty,$$

where $||y||_\infty = \max\limits_{x \in [0,\, l]} |y(x)|$.

Using an extension of the Prüfer type transformation (see [4]), in paper [1] were constructed the sets S_k^ν, $k \in \mathbb{N}$, $\nu \in \{+, -\}$, of functions of E that have the oscillation properties of eigenfunctions of the linear eigenvalue problem

$$\begin{cases} (\ell y)(x) = \lambda\tau(x)\, y(x), \ x \in (0, l), \\ y(0) = y'(0) = y(l) = y'(l) = 0, \end{cases}$$

and their derivatives [1,§ 3.1].

It follows from [15, Theorem 3.3] that half-linear eigenvalue problem

$$\begin{cases} (\ell y)(x) = \lambda\tau(x)\, y(x) \ + \alpha(x) y^+(x) \ + \beta(x) y^-(x), \ x \in (0, l), \\ y(0) = y'(0) = y(l) = y'(l) = 0, \end{cases} \tag{2.1}$$

has two infinitely increasing sequences $\{\lambda_k^+\}_{k=1}^\infty$ and $\{\lambda_k^-\}_{k=1}^\infty$ of simple half-eigenvalues such that

$$\text{if } k' > k \geq 1, \text{ then } \lambda_{k'}^{\nu'} > \lambda_k^\nu \text{ for each } \nu,\, \nu' \in \{+, -\}.$$

Moreover, for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ the half-eigenfunction $y_{k,\nu}$ corresponding to the half eigenvalue λ_k^ν of problem (2.1) lies in S_k^ν.

We introduce the following notations (see [9, p. 30]):

$$N_\alpha = \max_{x \in [0, l]} |\alpha(x)|, \ N_\beta = \max_{x \in [0, l]} |\beta(x)|, \ \tau_0 = \min_{x \in [0, l]} \tau(x),$$

$$I_k^+ = \left[\lambda_k^+ - \frac{N_\alpha + N_\beta + M}{\tau_0}, \lambda_k^+ + \frac{N_\alpha + N_\beta + M}{\tau_0}\right],$$

$$I_k^- = \left[\lambda_k^- - \frac{N_\alpha + N_\beta + M}{\tau_0}, \lambda_k^- + \frac{N_\alpha + N_\beta + M}{\tau_0}\right].$$

Remark 2.1. If the function $g \in C\left([0, l] \times \mathbb{R}^5; \mathbb{R}\right)$ satisfies the condition

$$g(x, y, s, v, w, \lambda) = o(|y| + |s| + |v| + |w|) \text{ as } |y| + |s| + |v| + |w| \to 0, \tag{2.2}$$

uniformly in $x \in [0, l]$ and in $\lambda \in \Lambda$, for any bounded interval $\Lambda \subset \mathbb{R}$, then we consider the bifurcation from zero, i.e. the existence of solutions to problem (1.1), (1.2) having an arbitrarily small norm. Note that the bifurcation from zero of nontrivial solutions to problem (1.1), (1.2) was studied in [2] in the case of $f \equiv 0$, and in [9] in the case of $f \not\equiv 0$. These papers studied not only the existence of nontrivial solutions with sufficiently small norms, but also the existence of nontrivial solutions with sufficiently large norms. In the case of $f \equiv 0$ it was shown that for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ there exists a continuum C_k^ν of solutions of problem (1.1), (1.2) that contains $(\lambda_k^\nu, 0)$, contained in $(\mathbb{R} \times S_k^\nu) \cup \{(\lambda_k^\nu, 0)\}$ and unbounded in $\mathbb{R} \times E$. In the case of $f \not\equiv 0$ it was proven that for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ there exists a component D_k^ν of solutions of problem (1.1), (1.2) that contains $I_k^\nu \times \{0\}$, contained in $(\mathbb{R} \times S_k^\nu) \cup \{(I_k^\nu \times \{0\})\}$ and unbounded in $\mathbb{R} \times E$.

Remark 2.2. Since condition (1.4) is satisfied, we consider the bifurcation from infinity, i.e., the existence of solutions to problem (1.1), (1.2) having an arbitrarily large norm. If, in addition (1.4), $f \equiv 0$, then the problem is called asymptotically linear and the existence of solutions with sufficiently large norms bifurcating from infinity is discussed in [8]. We recall that in [8] for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ it is established the existence of a continuum C_k^ν of nontrivial solutions to problem (1.1), (1.2) with $f \equiv 0$ containing (λ_k^ν, ∞) and neighborhood Q_k^ν of the point (λ_k^ν, ∞) such that $(\mathcal{C}_k^\nu \cap \mathcal{Q}_k^\nu) \subset \mathbb{R} \times S_k^\nu$ and either $\mathcal{C}_k^\nu$ meets $(\lambda_{k'}^{\nu'}, \infty)$ with respect to the set $\mathbb{R} \times S_{k'}^{\nu'}$ for some $(k', \nu') \neq (k, \nu)$, or $\mathcal{C}_k^\nu \backslash Q_k^\nu$ meets $\{(\lambda, 0) : \lambda \in \mathbb{R}\}$ for some $\lambda \in \mathbb{R}$, or the natural projection $P_{\mathbb{R}}(\mathcal{C}_k^\nu)$ of the set $\mathcal{C}_k^\nu$ onto $\mathbb{R} \times \{0\}$ is unbounded. If $f \not\equiv 0$, then in next section we show that the bifurcation occurs from some intervals of the line $\mathbb{R} \times \{\infty\}$. Moreover, we will study global character of the union of all components of the set of nontrivial solutions bifurcating from these intervals.

3. Global bifurcation from infinity of nontrivial solutions of problem (1.1), (1.2)

To study the global bifurcation of solutions to problem (1.1), (1.2) with condition (1.4) we will use the approach used in [3, 11, 13], which consists in transforming the problem of bifurcation from infinity to a problem involving bifurcation from zero.

Let $\mathcal{C}$ be the set of nontrivial solutions of problem (1.1), (1.2). If $(\lambda, y) \in \mathcal{C}$, then dividing (1.1), (1.2) by $||y||_3^2$ and setting $v = \frac{y}{||y||_3^2}$ we get

$$\begin{cases} \ell(v) = \lambda\tau(x)v + \alpha v^+ + \beta v^- + \frac{f(x, y, y', y'', y''', \lambda)}{||y||_3^2} + \frac{g(x, y, y', y'', y''', \lambda)}{||y||_3^2}, \ 0 < x < l, \\ v(0) = v'(0) = v(l) = v'(l) = 0. \end{cases} \tag{3.1}$$

Note that $||v||_3 = \frac{1}{||y||_3}$, and consequently, $||y||_3 = \frac{1}{||v||_3}$ and $y = \frac{v}{||v||_3^2}$. We define the functions $\hat{f}(\lambda, v)$, $\hat{g}(\lambda, v) \in C[0, l]$ as follows:

$$\hat{f}(\lambda, v)(x) = \begin{cases} ||v||_3^2 f\left(x, \frac{v(x)}{||v||_3^2}, \frac{v'(x)}{||v||_3^2}, \frac{v''(x)}{||v||_3^2}, \frac{v'''(x)}{||v||_3^2}, \lambda\right) & \text{if } v(x) \neq 0, \\ 0 & \text{if } v(x) = 0, \end{cases} \tag{3.2}$$

$$\hat{g}(\lambda, v)(x) = \begin{cases} ||v||_3^2 g\left(x, \frac{v(x)}{||v||_3^2}, \frac{v'(x)}{||v||_3^2}, \frac{v''(x)}{||v||_3^2}, \frac{v'''(x)}{||v||_3^2}, \lambda\right) & \text{if } v(x) \neq 0, \\ 0 & \text{if } v(x) = 0. \end{cases} \tag{3.3}$$

Then problem (3.1) reduces to the following equivalent form

$$\begin{cases} \ell(v) = \lambda\tau(x)v + \alpha(x)v^+ + \beta(x)v^- + \hat{f}(\lambda, v) + \hat{g}(\lambda, v), \ 0 < x < l, \\ v(0) = v'(0) = v(l) = v'(l) = 0. \end{cases} \tag{3.4}$$

Obviously, for each $\lambda \in \mathbb{R}$ the pair $(\lambda, 0)$ is a trivial solution to problem (3.4). By (3.2) it follows from conditions (1.3) that

$$||\hat{f}(\lambda, v)||_\infty = ||v||_3^2 \left\| f\left(x, \frac{v}{||v||_3^2}, \frac{v'}{||v||_3^2}, \frac{v''}{||v||_3^2}, \frac{v'''}{||v||_3^2}, \lambda\right)\right\|_\infty \leq$$
$$M||v||_3^2 \left\| \frac{v}{||v||_3^2} \right\|_\infty = M\, \|v\|_\infty\,, \tag{3.5}$$

In view of condition (1.4), by [1, Lemma 5.5] for any sufficiently small $\varepsilon > 0$ there exists sufficiently large $\Delta_\varepsilon > 0$ such that

$$|g(x, y, y', y'', y''', \lambda)| < \varepsilon ||y||_3 \text{ for } \lambda \in \Lambda \text{ and } ||y||_3 > \Delta_\varepsilon. \tag{3.6}$$

Let $\delta_\varepsilon = \frac{1}{\Delta_\varepsilon}$ and $\lambda \in \Lambda$, $||v||_3 < \delta_\varepsilon$. Then we have $||y||_3 = \frac{1}{||v||_3} > \frac{1}{\delta_\varepsilon} = \Delta_\varepsilon$, and consequently, by (3.6) we get

$$\frac{||\hat{g}(\lambda, v)||_\infty}{||v||_3} = \frac{||v||_3^2 \left\| g\left(x, \frac{v}{||v||_3^2}, \frac{v'}{||v||_3^2}, \frac{v''}{||v||_3^2}, \frac{v'''}{||v||_3^2}, \lambda\right)\right\|_\infty}{||v||_3} =$$
$$\frac{\left\| g\left(x, \frac{v}{||v||_3^2}, \frac{v'}{||v||_3^2}, \frac{v''}{||v||_3^2}, \frac{v'''}{||v||_3^2}, \lambda\right)\right\|_\infty}{\frac{1}{||v||_3}} = \frac{\|g(x, y, y', y'', y''', \lambda)\|_\infty}{||y||_3} < \varepsilon,$$

which implies that

$$||\hat{g}(\lambda,\, v)||_\infty = o(||v||_3), \quad \text{as } ||v||_3 \to 0, \tag{3.7}$$

uniformly for $\lambda \in \Lambda$.

Thus by (3.5) and (3.7) the transformation

$$H : (\lambda, y) \to (\lambda, v)$$

turns a "bifurcation at infinity" problem (1.1), (1.2) into a "bifurcation from zero" problem (3.4).

Let $\hat{\mathcal{C}}$ we denote the set of nontrivial solutions to problem (3.4). By construction, the transformation H maps $\mathcal{C}$ into $\hat{\mathcal{C}}$ and, heuristically, interchanges points at $y = 0$ (respectively, $y = \infty$) with points at $y = \infty$ (respectively, $y = 0$).

Remark 3.1. It follows from [9, Lemma 2.2] that for each $k \in \mathbb{N}$ and each $\nu \in \{+\,, -\}$ the set of bifurcation points to problem (3.4) with respect to the set $\mathbb{R} \times S_k^\nu$ is nonempty. Moreover, if $(\lambda, 0)$ is a bifurcation point of this problem with respect to the set $\mathbb{R} \times S_k^\nu$, then $\lambda \in I_k^\nu$.

Remark 3.2. By the above arguments it follows from Remark 3.1 that for each $k \in \mathbb{N}$ and each $\nu \in \{+\,, -\}$ the set of bifurcation points to problem (1.1), (1.2) with respect to the set $\mathbb{R} \times S_k^\nu$ is nonempty. Moreover, if (λ, ∞) is a bifurcation point of this problem with respect to the set $\mathbb{R} \times S_k^\nu$, then $\lambda \in I_k^\nu$.

For each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ by $\hat{\mathcal{D}}_k^\nu$ we define the union of all the components of $\hat{\mathcal{C}}$ which meet $I_k^\nu \times \{0\}$ with respect to $\mathbb{R} \times S_k^\nu$. Using Remark 3.1 and by following the arguments of Theorem 3.1 on p. 151 of [13] we can show that $\hat{\mathcal{D}}_k^\nu$ is not empty.

Remark 3.3. Since condition (1.4) is satisfied, statement [1, Lemma 1.1] does not hold. For this reason, for problem (3.4), statement [9, Theorem 3.1] does not hold.

However, by extending the approximation technique from [5] and combining it with the global bifurcation results in [6], [9], [10] and [14] we can show that for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ for $\hat{\mathcal{D}}_k^\nu$ the following statements hold:

(a) there exists a neighborhood $\hat{\mathcal{Q}}_k^\nu$ of $I_k^\nu \times \{0\}$ in $\mathbb{R} \times E$ such that $\hat{\mathcal{D}}_k^\nu \bigcap \hat{\mathcal{Q}}_k^\nu \subset \mathbb{R} \times S_k^\nu$;

(b) either $\hat{\mathcal{D}}_k^\nu \bigcap \hat{\mathcal{D}}_{k'}^{\nu'} \neq \emptyset$ for some $(k', \nu') \neq (k, \nu)$, or $\hat{\mathcal{D}}_k^\nu$ is unbounded in $\mathbb{R} \times E$; if in this case $\hat{\mathcal{D}}_k^\nu$ is unbounded in $\mathbb{R} \times E$, then either $\hat{\mathcal{D}}_k^\nu$ meets $\mathbb{R} \times \{\infty\}$ for some $\lambda \in \mathbb{R}$, or natural projection $P_{\mathbb{R}}(\hat{\mathcal{D}}_k^\nu)$ of $\hat{\mathcal{D}}_k^\nu$ onto $\mathbb{R} \times \{0\}$ is unbounded.

Now for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ by $\mathcal{D}_k^\nu$ we define the union of all the components of $\mathcal{C}$ which meet $I_k^\nu \times \{\infty\}$ with respect to $\mathbb{R} \times S_k^\nu$. It is obvious that $\mathcal{D}_k^\nu$ is not empty and $\mathcal{D}_k^\nu$ is the inverse image $H^{-1}(\hat{\mathcal{D}}_k^\nu)$ of $\hat{\mathcal{D}}_k^\nu$ under the transformation H. Then, using statements (a) and (b) for the set $\hat{\mathcal{D}}_k^\nu$ and Remark 3.2, we obtain the following result on the global bifurcation from infinity for problem (1.1), (1.2).

Theorem 3.1. *For each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ the set $\mathcal{D}_k^\nu$ has the following properties:*

(i) there exists a neighborhood $\mathcal{Q}_k^\nu$ of $I_k^\nu \times \{\infty\}$ in $\mathbb{R} \times E$ such that $\mathcal{D}_k^\nu \cap \mathcal{Q}_k^\nu \subset \mathbb{R} \times S_k^\nu$;

(ii) either $\mathcal{D}_k^\nu \cap \mathcal{D}_{k'}^{\nu'} \neq \emptyset$ for some $(k', \nu') \neq (k, \nu)$, or $\mathcal{D}_k^\nu$ meets $\mathbb{R} \times \{0\}$ for some $\lambda \in \mathbb{R}$, or natural projection $P_{\mathbb{R}}(\mathcal{D}_k^\nu)$ of $\mathcal{D}_k^\nu$ onto $\mathbb{R} \times \{0\}$ is unbounded.

Next, if both conditions (1.4) and (2.3) are satisfied, then we can improve Remark 2.1 and Theorem 3.1 as follows.

Theorem 3.2. *If conditions (1.4) and (2.3) are satisfied, then for each $k \in \mathbb{N}$ and each $\nu \in \{+, -\}$ we have*

$$\mathcal{D}_k^\nu \subset \mathbb{R} \times S_k^\nu \;\; and \;\; \mathcal{D}_k^\nu \cap \mathcal{D}_{k'}^{\nu'} = \emptyset \;\; for\ any \;\; (k', \nu') \neq (k, \nu).$$

Moreover, if $\mathcal{D}_k^\nu$ meets $\mathbb{R} \times \{0\}$ for some $\lambda \in \mathbb{R}$, then $\lambda \in I_k^\nu$. In a similar way, if $\hat{\mathcal{D}}_k^\nu$ meets $\mathbb{R} \times \{\infty\}$ for some $\lambda \in \mathbb{R}$, then $\lambda \in I_k^\nu$.

The proof of this theorem is similar to that of [13, Theorem 3.3]

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